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Medal Day Proceedings at The Franklin Institute

Fundamental Particles
Hans Albrecht Bethe

VOL. 268 NO. 6

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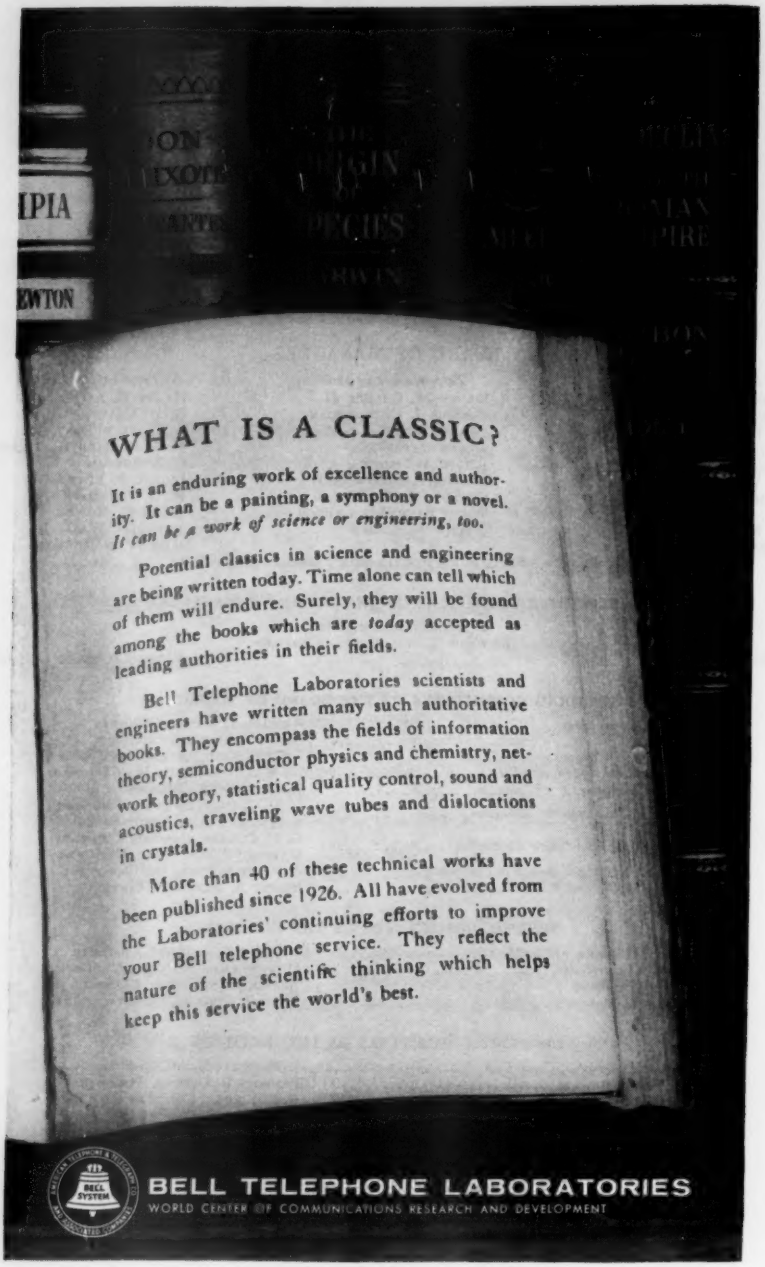
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COSMIC THUNDERSTORMS*

BY

C. E. R. BRUCE[1]

SUMMARY

Applications of the writer's electrical discharge theory of some astrophysical phenomena are discussed, and interesting inter-relationships are adduced between corresponding physical processes in the laboratory and in the terrestrial, stellar and galactic atmospheres. The building-up of electrostatic fields in these atmospheres is discussed, and the breakdown of these fields in electrical discharges is shown to account for the light emission from, and gas movements in, the atmospheres of the long-period variable and combination-spectra stars. The theory has a bearing on the evolutionary process in, and chemical composition of, late-type stars. It will explain the gas movements observed in extra-galactic radio sources, and accounts for the magnetic fields and "relativistic" electrons required by the synchrotron theory of the radio noise itself, for which no other explanation has so far been offered. The theory likewise suggests an explanation for the existence in some galaxies of two stellar populations, which is in agreement with observations of some of their major features. A new theory of propagation of these cosmical electrical discharges is put forward which offers a way out of the difficulty hitherto met in explaining the short time lags of some magnetic storms on the causative solar outbursts, and the correspondingly high average velocities of the particles responsible for these storms. These are much greater than any velocities so far observed at or near the sun's surface. It is shown that in these large cosmic electrical discharges thermonuclear reactions become important when the discharge temperature reaches about 400,000,000 K.

INTRODUCTION

Some years ago the writer (1a)[2] attempted to out-Franklin Franklin in the extension of the field of electrical discharges in gases, by suggesting a series of steps, the greatest of which may be as great as the universe itself. However, the present survey will be limited to the presentation of the evidence for some applications of the theory on the stellar and galactic scales. The manifestations of a series of physical processes

* Based on a series of Reports of the Electrical Research Association.

[1] The Electrical Research Association Laboratory, Leatherhead, England.

[2] The boldface numbers in parentheses refer to the references appended to this paper.

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

will be studied in the laboratory, as well as in the terrestrial, stellar and galactic atmospheres, in the hope that the consideration of electric field-building and discharge phenomena on such a wide variety of magnitudes may prove suggestive for meteorological and nuclear physicists, as well as for astrophysicists and those interested in the study of electrical discharges themselves. For, in the course of these investigations an estimate has been obtained for the temperature required for the engendering of thermonuclear reactions to quite a marked degree in these extensive electrical discharges in cosmic atmospheres. This is found to occur at a temperature of about 400 million degrees absolute.

ATMOSPHERIC ELECTRIC FIELD-BUILDING

In a letter to Dr. Lining of Charles Town, South Carolina, addressed and dated "Philadelphia, March 18, 1755," Franklin wrote: "I wish I could give you any satisfaction in the article of clouds. I am still at a loss about the manner in which they become charged with electricity; no hypothesis I have yet formed perfectly satisfying me." After over 200 years that last sentence might, and indeed can still be found in any exhaustive discussion of the subject. For example a paper presented to last year's U. S. Air Force Conference on Atmospheric Electricity and entitled "The Lightning Mechanism and its Relation to Natural and Artificial Freezing Nuclei" opens with the sentence, "There is as yet no generally accepted theory for the electric charge generation in thunderstorms," while another paper refers to "the unsolved problem of thunderstorm electricity."

Not surprisingly it is still more difficult to deal adequately with the problem of charge separation and field-building in cosmic atmospheres, in which the air, water and ice of the terrestrial atmosphere are replaced mainly by hydrogen and helium and the oxides, hydrides etc. of a variety of metals, such as titanium, zirconium, vanadium, etc. Indeed the writer has often been told authoritatively, as at the Liège astrophysical symposium in 1957, that it is "impossible" for electrostatic fields to be built up, even in the relatively cold atmospheres of the long-period variable stars. However he hopes to show that far from it being "impossible," it would be quite surprising if electrical effects were *not* observed in the conditions existing in these stellar atmospheres.

Terrestrial Atmospheric Electric Fields

Two names which should be better known to students of electricity than they appear to be are those of Stephen Gray and the late Professor P. E. Shaw of Nottingham University. The former first showed that electricity could be conducted, and thus greatly extended the science of electricity as it was known in 1729 (2), while the latter (3a) fundamentally changed the subject of electrostatics by showing that in order

to cause the separation of electric charges by the rubbing together of two bodies it is not necessary to start with two different materials, an experimental fact which still causes surprise to most physics students when they are informed of it. Two sticks of the same material will become oppositely charged provided the rubbing is asymmetrical; for example, if a limited section, say 1 cm. in length, of one rod, is rubbed along the whole length of a similar rod, then the two rods will have opposite charges.

Such asymmetrical reactions obviously occur in wind-blown dusts and powders, and Shaw showed (3b) that these also become charged, even though the reactions are limited to those between particles of the same material. Furthermore, he showed that the charging effect is of the same order of magnitude with cold dry ice particles as it is with sand.

In view of the chief cause of asymmetry of the effects in these conditions we may suppose that on an average larger and smaller particles will become oppositely charged, and there is some experimental evidence to support this conclusion. Their separation in wind-blown clouds of dust in a gravitational field will then lead to the generation of electric fields in such clouds. It is well known that electric fields are set up in such circumstances, and in terrestrial sand and dust storms and in the ejectamenta from volcanoes the fields so generated can lead to the electrical breakdown of air at atmospheric pressure.

It seems to the writer significant that during a discussion of thunderstorm problems at the Royal Meteorological Society (4), two of the most active observers both averred that no electrical effects are to be anticipated in thunderclouds until the anvil-shaped cap of cirrus cloud is formed at the top of the thundercloud. This forms at about − 30 C. and at a height of 30,000 to 40,000 ft., and is composed of dry ice crystals. This view of the critical requirement for the occurrence of electrification in thunderclouds is supported by the recent mass attack on this problem in the U. S. (5). It was found that lightning only occurs when the top of the thundercloud reaches heights of the order of 30,000 to 40,000 ft. and temperatures below − 20 C. Though the actual physical processes involved in thundercloud charge separation are still the subject of considerable discussion, it seems to the writer that these observations in the laboratory, in sand and dust storms, and in volcanic eruptions point to the adequacy of static electrification to explain the phenomena (3b).

This is supported by other papers in the aforementioned volume of the proceedings of the second U. S. Air Force on Atmospheric Electricity where Chalmers (6) writes ". . . there seems to be support for the idea that the charge separation is concerned with ice particles colliding with one another," as Simpson and Scrase had earlier suggested. In an investigation of charge generation on a mountain top it was noted that "All strong charging rates are connected with ice crystals in the air (7)."

But perhaps the strongest evidence on the origin of thunderstorm electricity afforded by that Symposium is the observation of the quite remarkable intensity of the electrical effects in the electric storms associated with tornadoes and at heights where ice particles alone exist (8).

Electric Fields in Stellar Atmospheres

The most obvious extension of these ideas is to the atmospheres of the long-period variable stars, the outstanding characteristics of which, apart from their great cyclical variation in optical magnitude, are their size and their extensive atmospheres, and their very low temperatures; some of them hardly shine at all, and the highest of their "surface" temperatures is under 4000 K. These cold "surfaces"—if they can be said to have a surface at all—have radii approximating in some cases to that of the Earth's orbit, and outside these "surfaces" extend tenuous atmospheres which could in some cases envelop the whole solar system.

These atmospheres would be, and are, relatively cold, apart from the periodical outbursts, during which the nature of the light emitted shows that some of it must originate in gas whose temperature has somehow been raised to 5000 or 10,000 K., and in a few cases even to 500,000 K. or a million degrees absolute. The vexed question has been, whence come these high temperatures? To which the writer's reply is, from lightning flashes in stellar thunderstorms (1a, c). For at minimum light the temperatures of these extensive atmospheres fall far below their "surface" temperatures of 1500 or 2000 K., and there would appear to be nothing to prevent them reaching values at which the electrical conductivity is sufficiently low to allow of the generation of electrostatic fields.

At these low temperatures a number of materials, such as metallic oxides, hydrides, and carbides will solidify out of the atmosphere. The existence at minimum phase of these solid or liquid particles had indeed already been deduced, as they offer the likeliest explanation of a large proportion of the diminution of the star's light at minimum brightness. To a large extent the nature of the light remains the same—there is just less of it. It is veiled by the cloud of particles.

It is also known from a spectroscopic analysis of the light that great winds blow in these atmospheres, with velocities up to more than 10 km. per second, so that the solid particles in these atmospheres will be subject to the violent impacts required for the generation of static.

The conclusion would appear to be inevitable that there will be considerable generation of static and of electric fields in these stellar atmospheres. These fields will go on building up at an increasing rate as the temperature falls towards minimum, so that, unless some other, and hitherto quite unforeseen, cause of the outburst becomes effective, electrical breakdown in discharges is bound to occur sooner or later.

Temporal Characteristics of Stellar Outbursts

One can compare very roughly the time which will be required for the build-up to breakdown by comparing the gas densities and velocities and the gravitational forces in these stellar atmospheres with those observed in thunderstorms. Whereas the build-up time in the thundercloud is of the order of 100 seconds, the estimated time under these stellar atmospheric conditions is of the order of 10^6 to 10^9 seconds, according as the process of charge separation depends on the first or second power of the relative velocity of the particles (1c). This agrees as well as can be expected with the observed periods of these stars, which range from about 100 to 600 days, or 10^7 to 10^8 seconds. The writer has therefore suggested (1d) that meteorological physicists may be able to elucidate the process of charge separation in thunderclouds, by a more precise comparison of the conditions therein, with those existing in the different types of long-period variable and combination-spectra stars, to which more reference will be made later.

Another check on the times involved in these stellar outbursts is obtained from a consideration of the duration of the period during which bright emission lines are observed in the stars' spectra, indicating the occurrence of the discharges. Apart from an effect to be discussed later, which does not affect the present argument, the velocity of propagation of electrical discharges will be independent of the gas density, and equal to the velocity of propagation of the lightning leader stroke at atmospheric pressure—that is, 10^7 to 10^8 cm. per second, the velocity of propagation of electrical breakdown in a hydrogen atmosphere being probably slightly greater than in air (9). Since the distances involved are of the order of 10^{14} to 10^{15} cm., the duration of the discharge process will be of the order of 10^7 seconds, again in good agreement with the observed periods of variation, during about half of each of which the bright lines are observed in these stellar spectra. Thus the temporal characteristics of this type of star agree reasonably well with those to be expected on the "thunderstorm" theory of their periodical outbursts.

LIGHT EMISSION FROM LONG-PERIOD VARIABLES

The general nature of the light itself during each increase in magnitude of these stars—which at maximum may reach 10,000 times their brightness at light minimum, the average increase being by a factor of about 100—and its regular "programme," is also quite in accord with the electrical discharge theory (1c). Indeed it has so far proved impossible to account for it in any other way. For in this low temperature atmosphere, mainly hydrogen, there suddenly appear emission lines of hydrogen, helium, including ionized helium, and ionized metals. As we shall see later, in the closely associated combination-spectra stars,

the level of excitation reaches that of six times, and even possibly nine or thirteen times, ionized iron atoms, representing an excitation equivalent to temperatures of between five hundred thousand and a million degrees, or more. Indeed so varied is the light emitted by this last type of star at different phases of its cycle, that they have been assumed to comprise a pair of stars, one very cold and one very hot, and an associated nebula.

However, we shall consider simply one large cold star surrounded by an extensive atmosphere. The rate of build-up of the electric field increases with the square of the density, with the gravitational force, and with the velocity; while the breakdown voltage is inversely proportional to the gas density. It follows (1c) that the conditions requisite for electrical breakdown will be reached first low down in the star's atmosphere and the discharges will be propagated outwards towards the star's peripheral layers.

The writer has emphasized (1b, e) that these long electrical discharges will serve as "energy pumps," so to speak, just as does the lightning leader stroke, causing energy generated in one place to be liberated at another. In the lightning discharge, for example, whereas the electrical energy generator is in the thundercloud, the highest current in the discharge actually flows just at the Earth's surface (1f), several kilometers away from the generator. The leader stroke acts as almost a complete short-circuit of the space between the cloud and the ground (1g, h), so that very high field concentrations occur around its advancing tip. This effect is likely to be enhanced in an atmosphere in which the discharge is propagated outwards through a decreasing gas density.

There are thus two effects to be looked for as the discharge proceeds. In the early stages, since it starts low down in the star's atmosphere, the light will be subject to considerable general and selective absorption by the dust particles and molecules of the vapors which abound, such as the oxides of titanium, zirconium, vanadium, etc., as well as C_2, CN, and other radicals. It is not to be expected, therefore, that such series of emission lines as the Balmer series of hydrogen, or the various multiplets in say the iron spectrum, will have the relative intensities observed in the laboratory, or anything like them. These relationships will be considerably mutilated by differential absorption in the upper regions of the atmosphere. However, as the discharge is propagated outwards, and as the energy liberated in it causes dissociation of the molecules already referred to, then the relative intensities of these series and multiplets will approach more and more those observed in the laboratory.

This sequence of events has been observed repeatedly by Merrill and other investigators. It is well illustrated, for example, by the variation of the individual lines of multiplet (2) of Fe of which Merrill writes

(10a): "In the last column, phase + 162 days (i.e. 162 days after maximum light) the relative intensities are the same as in the laboratory. At earlier phases, the intensities are modified, probably by TiO band absorption, as in R Leonis. The behaviour of this multiplet presents another example of the general tendency of bright lines to escape from the effects of the reversing layer as the phase advances." The same explanation will account for the wide variation in the relative intensities of $H\gamma$:$H\delta$ at different phases of the brightness cycle (10b). It is only some considerable time after maximum light that this ratio approaches the value observed in the laboratory.

COMBINATION-SPECTRA SPECTRA STARS

As a result of the intensification of the field at the head of the advancing discharge, and its projection outwards into regions of lower gas pressure, referred to above, the excitation of the gas will be increased. The spectrum of the gas will, therefore, change during the outburst from one of high temperature at relatively high gas pressure, to one of higher excitation at lower gas density, with forbidden lines entering as the very low pressures of the outer regions of the star's atmosphere are reached. The former of these two phases accounts for the spectrum to account for which it was earlier assumed that the large cold star was accompanied by a small hot "companion" star; while the later stages of the discharge account for the "nebular" contribution to the spectrum. It was therefore suggested (1c) that the theory will account for the combination spectra in such stars as R Aquarii, Z Andromedae, BF Cygni and AX Persei. In these the initial bright line spectrum, that usually attributed to the postulated "companion" star, comprising lines of H, HeI, Fe II, Ti II, and Si II, gives place, after 100 or 200 days, to a spectrum of higher excitation, containing lines of He II, N III, C III, [O III], [Ne III] and [Fe III]. The nature of this last nebular spectrum is in accord with the suggestion that it originates in regions of very low pressure, far out in the star's atmosphere, towards the completion of electrical neutralization.

The two spectra follow one another fairly regularly after periods of the order of 100 to 200 days in different stars. One observer (11) summed up his description of the sequence of the two different types of spectra by concluding that it is just ". . . as though they occurred as a consequence of the propagation of running waves over an extended medium." This will be seen to be in accord with the electrical discharge theory, the "running waves" being waves of electrical excitation.

CONDITIONS FOR THE INITIATION OF LONG ELECTRICAL DISCHARGES

Though the period of variability and brightness at maximum of the long-period variable stars are fairly well defined, they are subject to considerable variation in any one star. This may amount to about 10

days in a period of say 200 days, and to one or two magnitudes in maximum brightness. This variability may have an interesting analogy in the variability of the current in different lightning flashes in the same thunderstorm.

Some years ago (1g), when putting forward a new theory of the initiation and propagation of lightning leader strokes, the writer showed that the theory would explain the very wide variation in lightning currents. For a lightning flash to occur two things are necessary: first, an average electric field between the two charges in the cloud, or between one of these charges and its image in the Earth, sufficient to maintain the process of arc conduction in the leader stroke, when once it is initiated, that is, an average field of 10 to 100 V/cm; second, in that relatively low average field there must exist a field concentration, such as is caused by a tall grounded building on the Earth, or an elongated volume of space charge in the cloud, sufficient to cause the transition from the field-maintained corona discharge in the St. Elmo's fire at its tip, to a thermally-ionized column of arc discharge. When this transition occurs it was shown that the discharge will become self-propagating, so to speak, and bridge the gap. The smaller the initiating field concentration, the greater must be the average field before the leader stroke is initiated, and the greater will be the current in the discharge when it does occur.

It may be noted in passing that this new conception had an important bearing on the theory of the operation of a lightning conductor (1g), probably the first major change since its introduction by Franklin nearly two hundred years earlier. For the field concentration at the advancing tip of the leader stroke will also vary with the average pre-discharge field, so that upward streamers will be initiated from grounded buildings earlier in the leader stroke's descent for high average pre-discharge fields. Thus, heavy flashes will be attracted to the conductor from much greater lateral distances than will light or low current flashes. Previously it had been considered that the protective range of a lightning conductor depended only on its height, and not at all on the nature of the lightning flash.

Fortuitous variation in the distribution of space charge may be expected to cause a similar variability in the conditions required for discharge initiation in all long and purely atmospheric discharges. The longer the initiation of the discharge is delayed in the stellar atmosphere, the greater will be the cooling of the atmosphere after the previous outburst, and the greater also will be the amount of dust solidified out of the atmosphere during the minimum phase. This will have two results. It will cause a greater dimming of the star's light, and hence a lower light minimum, and there will also be a greater increase in the average pre-discharge field, and hence a greater outburst and increase in luminosity when the discharges do occur.

Thus, besides accounting for the irregularity of the periods and the amplitudes of the variations of brightness observed in these stars generally, the theory would also explain some observations made by Merrill (10c) on the combination-spectra star R Aquarii. He has pointed out that in a series of pulsations of this star in the early 1930's very marked dimming of the main "cold" red star was associated with extra bright outbursts of the "companion" star or discharge spectra. The idea of a "companion" star was introduced, as we have seen, to account for the early stages of the electrical discharge. Merrill is the leading observer of and authority on this type of variable star, and it should be recorded that, though the belief is generally held that in all cases two stars and a nebula are required to account for the phenomena, Merrill himself in his Monograph (10c) and papers has been careful to emphasize that in many cases, including R Aquarii itself, there is no positive evidence for the existence of the "companion" star as he has usually so written it, and that all might in fact come from one large "cold" star and its atmosphere. Summing up the discussion of this type of star in his Monograph, Merrill wrote (10d) that ". . . it would be inadvisable at the moment to accept without reserve the hypothesis of actual duplicity for all combination spectra."

EVOLUTION AND CHEMICAL COMPOSITION OF LATE-TYPE STARS

The application of the discharge theory to the long-period variable stars has a bearing on two questions of major interest in astronomy, namely, stellar evolution, and the uniformity of the chemical composition of matter throughout the universe, since the atmospheres of these late-type stars are one of the few places where there is generally considered to be a departure from this uniformity. The theory suggests that the observations can be explained by differences in the physical state of matter of the same general chemical composition.

Stars can be arranged in a series having decreasing "surface" temperatures until temperatures of the order of 3500 to 4000 K. are arrived at, that is, temperatures at which the freezing out of hydrides and carbides and carbon itself from their atmospheres begins. Below these temperatures the series apparently trifurcates, the three branches being differentiated by the different absorbing molecules in their atmospheres. The spectra of one group, the carbon stars of Classes R and N, show the bands produced by the C_2 and CN molecules; another group, the titanium stars of Class M, shows mainly bands of titanium oxide; while in the third group, the zirconium stars of Class S, the titanium oxide bands are replaced by those of zirconium oxide. These differences are usually attributed to actual differences in the chemical constitution of the stars' atmospheres. The writer, however, has suggested (1j) that the difference is mainly one of average physical state of the atmospheric material external to the star's "surfaces" or photospheres.

Let us assume that during the evolutionary process the average temperature of this outer atmosphere is falling. The arguments adduced can be reversed if in fact this average temperature rises as the stars age. The first molecules to form will be those of C_2 and CN. At still lower temperatures particles of carbon, carbides and hydrides will be formed so that the molecules of C_2 will disappear, and with them the C_2 bands will disappear from the spectrum. They will be replaced by the bands of molecules which associate at lower temperatures, such as zirconium oxide, which will begin to appear at temperatures of the order of 3000 K. However, in its turn zirconium oxide will freeze out and become solid particles at temperatures of around 2500 K., its place being taken by titanium oxide and others which associate at around these temperatures. Titanium oxide will remain in the vapor state, and give rise to bands in the star's spectrum, until it too solidifies at temperatures of around 1600 K.

At any one point in the evolution of the outer atmosphere of a star, therefore, the oscillations in temperature of that atmosphere, due to the occurrence of the discharges followed by a period of cooling and electric field-generation, will occur within a given range. This range will be appropriate for the appearance in its spectrum of each of the three main sets of bands in turn as the average temperature falls. First, the carbon molecules will disappear, having become particles of carbon which will no longer be vaporized to any appreciable extent by the discharges when they occur. The carbon bands will be replaced by zirconium bands, until they too in turn disappear when the average temperature falls so low that the solid particles of zirconium oxide are no longer vaporized, and finally only titanium oxide bands will appear in the star's spectrum throughout its cycle.

There is one piece of evidence which strongly supports the new theory, and which would appear entirely to negative the possibility that the explanation lies in differing chemical constitutions. It is quite possible on the view now proposed that stellar atmospheres will exist in which at minimum only titanium oxide bands appear in their spectra, but in which the rise in temperature caused by the discharges is so great that, at maximum, all the titanium oxide molecules are dissociated, and sufficient zirconium oxide particles are vaporized, to lead to the replacement of the titanium oxide bands by those of zirconium oxide at maximum brightness. In other words, the star will change from type M, at minimum, to type S, at maximum, a change which would be quite impossible if the difference between these two stellar types is one of chemical composition.

In fact, however, stars *do* exist in which this change occurs as the result of a specially great outburst—that is, when they reach what is for them an exceptionally bright maximum, and consequently an unusually high average temperature. One such star is χ Cygni. It has

been observed to change from type M to type S at unusually bright maxima.

It would thus seem that these three types of late stars—carbon stars, zirconium stars and titanium stars of types N, R, S and M, respectively—are not necessarily in conflict with the uniformity of chemical constitution of matter observed fairly generally throughout the universe, as they are generally believed to be, nor do they necessarily indicate a trifurcation of the stellar evolutionary sequence in the way they are generally regarded as doing.

GAS MOVEMENTS IN ELECTRICAL DISCHARGES

Perhaps the most intriguing inter-relationship so far brought to light between the characteristics of these electrical discharges in the laboratory, the atmosphere, and in stellar and galactic atmospheres, is that existing between the gas movements engendered by the discharges. We are not here concerned with movements analogous to the *explosive* movement of the *surrounding* gas, which results in the thunder of the lightning discharge. It is, in contrast, a *continuous* axial flow of the hottest gas *along* the *central regions* of the discharge channel. The latter acts like a hose-pipe squirting gas from regions of high current and high current density towards regions where the product of these two quantities is reduced.

The Arc and Lightning Discharges

R. C. Mason (12) showed that because the charged particles of the electric discharge flow along the channel in its own magnetic field, they will be constrained by the field to move inwards towards the axis of the discharge. He showed that this would result in an axial increase in gas pressure, which is proportional to the product of the current and the current density.

Maecker (13) later drew the "obvious" conclusion that constrictions in the discharge channel, such as exist at the anode and cathode spots of the arc discharge, will give rise to high pressures, and therefore to gas movement down the resulting pressure gradient. And so the anode and cathode jets of the electric arc were explained satisfactorily for the first time. King (14) has shown in these laboratories that these jets account in large part for the transfer of metal in the arc welding process, and explain why it is independent of gravity. (He has also shown that the temperature of the welding arc is several times greater than the 6000 or 7000 K. usually quoted for it. It is usually in the range 15,000 to 20,000 K.)

The pressure will increase with the product of the current and the current density, but the velocity of the gas flow cannot go on increasing indefinitely. It is limited by the velocity of sound in the gas at the temperature of the discharge. For example (1h), in the lightning dis-

charge the temperature will vary with the current in different flashes, but will almost certainly lie between 50,000 and 100,000 K., for periods of hundreds of microseconds or a millisecond. With these ranges of temperatures and times, the distance moved up the lightning channel by the gas and vaporized material at the Earth's surface will lie between 70 and 1000 cm. This agrees with, and indeed explains, the observations of Israel and Wurm (15) that metal lines are observed in the spectrum of a lightning flash up to a height of about 2 meters above the ground.

The Long-Period Variable Stars

The first extra-terrestrial application of these ideas is again to the discharges in the atmospheres of the long-period variable stars (1k). When the bright emission lines appear amid the molecular absorption bands in the spectra of these stars, they are those of ionized and neutral metal atoms, hydrogen, and helium, denoting gas temperatures of between 5000 and 10,000 K. Since the gas is largely ionized hydrogen the velocity of sound in it at these temperatures will lie between *8.5 and 12 km. per second.* This is an extremely narrow range of velocities when one considers that, apart from the theory now being put forward, the gas velocities might have been measured in miles per hour, miles per minute, miles, tens, hundreds or thousands or more of miles per second. However, extremely narrow though this theoretical range of gas velocities is, relative to the whole gamut of possible cosmic velocities, it contains both the average values obtained for these gas velocities by the two leading authorities on this type of star at Mount Wilson. In these stars the light absorption is so great that only that from the discharges on the near side of the star's atmosphere is photographed, so that the spectra show broadened emission lines displaced towards the violet relative to the absorption lines produced by the relatively stationary atmosphere. From the displacement of the emission lines towards the violet in the spectra of 72 long-period variable stars, Merrill (10e) obtained an average value for the velocity of the gas of *11 km. per second*, while from similar measurements in the spectra of seventeen closely similar irregular variable stars Joy (16) obtained an average velocity of *9 km. per second.*

The Combination-Spectra Stars

As has already been seen the combination-spectra stars are similar in many respects to the long-period variable stars. It was therefore somewhat disconcerting (11) from the point of view of this theory of these gas movements, to find that in one of these stars, AX Persei, Merrill (10f) had observed displacements of the emission lines relative

to the absorption lines which were equivalent to velocities of approach of 110 km. per second. Since the velocity of sound in a gas only increases as the square root of the absolute temperature, this meant that in the very extensive cold atmosphere of this star the gas temperature in the discharges must have reached 500,000 to 1,000,000 K., if the theory were to be saved. The theory *was* saved, however, by an equally surprising observation in another paper, by Swings and Struve (17), in which they showed that some of the emission lines in the spectra of AX Persei derived from Fe VI, Fe VII, and even possibly from Fe X, that is, from five, six, or even possibly nine times ionized iron atoms, which also require for their production the buffeting to be expected in a gas at the temperature of about a million degrees absolute, required to account for the high gas velocity.

Galactic Electrical Discharges

This upward trend of the axial temperatures with increase in the scale of these cosmic electrical discharges cannot go on indefinitely. There will come a time when those temperatures are reached, which are being eagerly pursued in the world's physical laboratories at the moment, namely those at which thermonuclear reactions will occur. When the latter are produced in sufficient degree then the increase in gas pressure which they produce will balance the inward pressure of the magnetic pinch effect, and further increase in temperature will be prevented.

Instead of taking place in deuterium, as in the laboratory discharges aimed at producing thermonuclear reactions, the cosmic discharges occur in a gas which is probably about 80 per cent hydrogen, with two parts in 10,000 deuterium, and with about 20 per cent helium and fractional percentages of the other atoms. The experts will probably agree that in the conditions of these large cosmic discharges temperatures of 10^8 to 10^9 K. will be required to cause nuclear reactions on a large scale. On the gas velocity thermometer, so to describe it, maximum velocities of 1750 to 5400 km. per second, the velocity of sound in ionized hydrogen at these temperatures, will therefore be observed in these larger electrical discharges when these temperatures are reached. These galactic discharges have indeed been investigated by Seyfert (18) at Mount Wilson, for he has examined the spectra emitted by bright emission patches in some extra-galactic nebulae. In these discharges velocities of recession are observed, as well as velocities of approach, so that the emission lines are broadened, rather than displaced. The velocities which Seyfert has recorded are in the range 1800 to 4250 km. per second, in good agreement with the above "theoretical" range of velocities.

COSMIC RADIO SOURCES

An interesting observation from the new point of view is Baade and Minkowski's (19) determination of the gas velocities in the well-known radio source, NGC 1275, illustrated in Fig. 1. They find that the gas in the well-defined arms is moving at a velocity of about 5250 km. per second, while that in the less well-defined patches of the back-

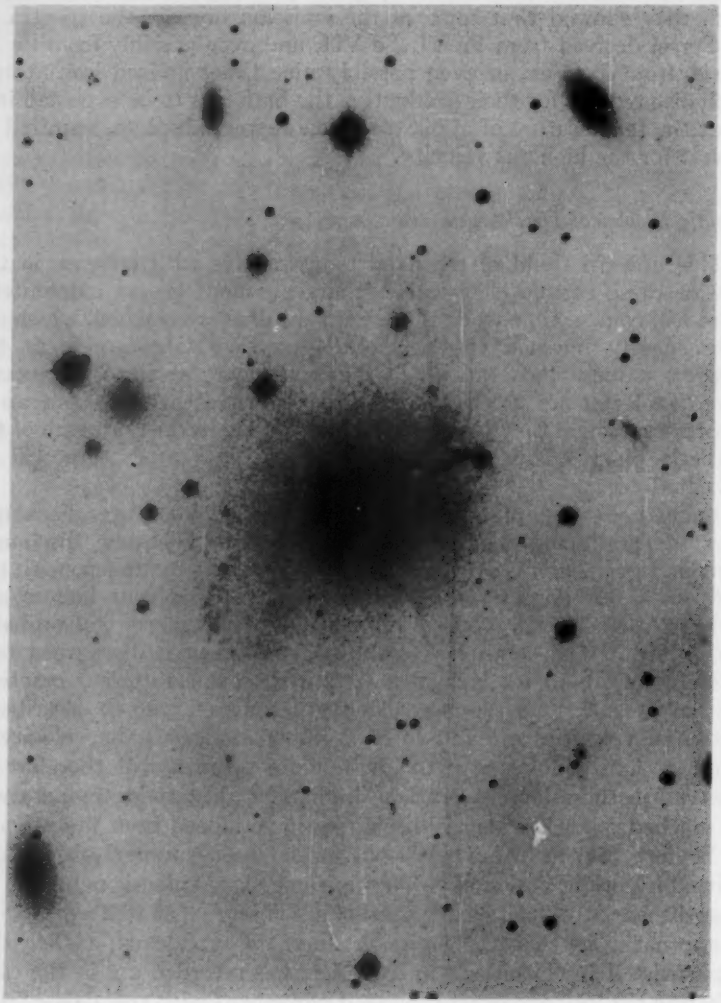

FIG. 1. Photograph of the radio source NGC 1275 taken with the 200-in. telescope at Mount Palomar Observatory. (λλ3600–5000 Å.)

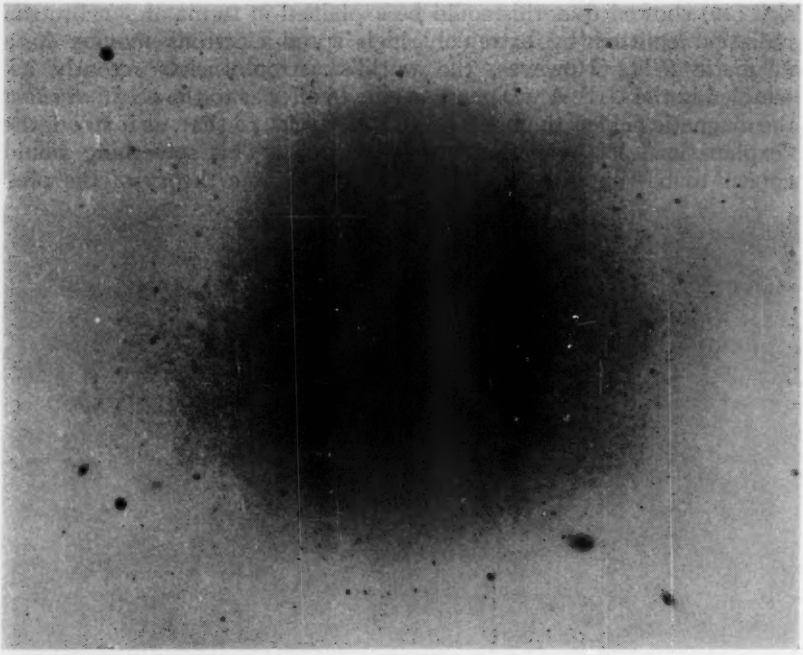

FIG. 2. Photograph of the radio source NGC 4486 taken with the 200-in. telescope at Mount Palomar Observatory. (λλ3600–5000 Å.)

ground gas is moving at about 8250 km. per second. They have therefore suggested that the source is a collision between two nebulae or galaxies, moving with these two velocities. The writer has suggested (1m) that at least some of these extra-galactic radio sources are galaxies in which the galactic radial electric field is breaking down and being neutralized in electrical discharges, which ultimately result in the formation of the spiral arms, for which last there is still no satisfactory theory. On this view the channels in NGC 1275 are these discharge channels, and the gas in them has been accelerated to a velocity of about 3000 km. per second in the line of sight by the pressure gradient caused by the magnetic pinch effect in the galactic discharge.

The light from the discharges on the other side of the galaxy may well be lost in the nebula's dusty atmosphere. The difficulty of photographing these discharge channels, even on the near side of a galaxy is illustrated in Figs. 2 and 3. The length of the discharge channel in that radio source, NGC 4486, is 300 parsecs and its diameter about 30 parsecs—a parsec being about 19 million million miles (19).

As regards the actual mechanism producing the radio waves, Shklov-

sky (20) showed that this could be explained in terms of synchrotron radiation emitted by extremely high speed electrons moving in a magnetic field. However, the world's astrophysicists recently assembled in the U. S. A. (21) had no clue to offer as to the origin of *either* the magnetic field *or* the "relativistic" electrons, so that, as it stood, the "explanation" left something to be desired. This something would appear to be supplied by the electrical discharge theory of the phe-

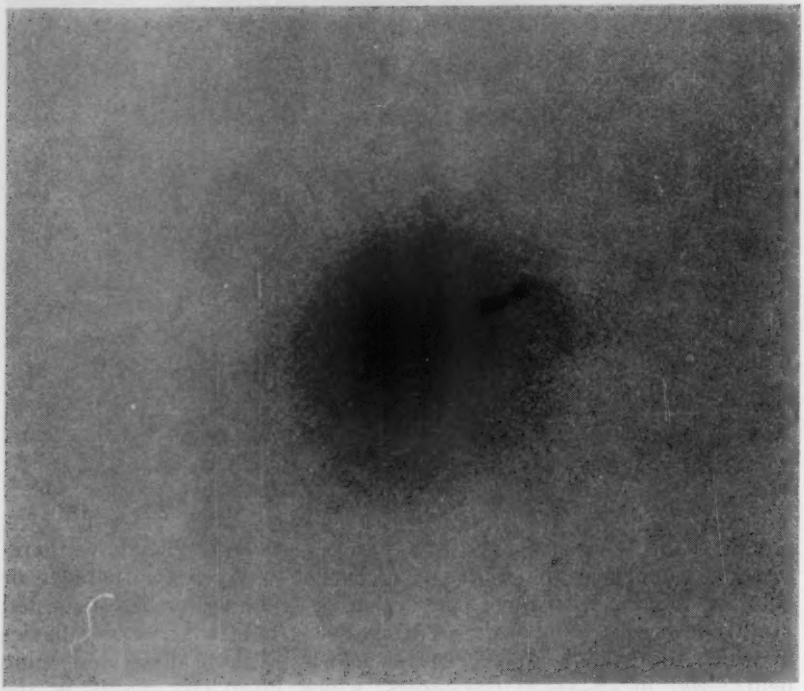

FIG. 3. Photograph of the central regions of NGC 4486 taken with the 100-in. telescope at Mount Wilson Observatory. ($\lambda < 4000$ Å.)

nomena. The current in the discharge obviously produces the required magnetic field. As regards the high speed electrons, the gas velocities being 1800 to 5400 km. per second, the corresponding electronic velocities will be over 40 times these values or over 7.2×10^9 to 2.16×10^{10} cm. per second. The theoretical values are therefore in the range required by Shklovsky's theory.

At an earlier Symposium of the I. A. U. (22) prominence was given to the prediction made by Shklovsky that the radiation from NGC 4486 should be polarized on the synchrotron radiation theory, and to Baade's observations confirming this prediction. However, many years ago (1p)

the writer pointed out that the radiation from these large single electrical discharges should be polarized, and that this could be looked for in the initial stages of novae, for example. As a result of this suggestion this observation was put on the observing program of Mount Wilson Observatory for the next bright nova outburst.

In the galactic radio source, the Crab Nebula, from which the radiation, both optical and radio, is similar to that from NGC 4486, the phenomena can be subjected to more detailed investigation. As a result of such an examination Woltjer (23) has recently deduced that the varying directions of the polarization can be accounted for if electric currents flow along the gaseous filaments. The conclusion that these filaments are electrical discharge channels would appear to be inevitable, and the observed gas velocity of over 1,000 km. per second enables their temperature to be determined as about 3×10^7 K.

Shklovsky (20) has estimated that if all metagalactic radio noise is to be accounted for as originating in such "jets" or discharge channels as that in NGC 4486, then at present about one per cent of all galaxies must be passing through this phase. From this the velocity of propagation of these discharges can be calculated, since, on the discharge theory, the time scale of these phenomena is determined by this velocity. The age of the nebulae is 10^9 to 10^{10} years, so that if at any one time one per cent are passing through a particular phase of their life, this phase must last for 10^7 to 10^8 years in any one nebula. As the length of the discharges is of the order of 10^4 to 10^5 light years, it follows that the velocity of propagation is of the order of 10^{-3} times the velocity of light, or of the same order as the velocity of propagation of electrical discharges in the terrestrial and stellar atmospheres. This is a result to be expected *a priori* on theory (1n), since the expression for the velocity of propagation of electrical breakdown depends on the product of the mean free path and the breakdown potential gradient. One is directly and the other inversely proportional to the gas density, so the velocity of propagation may be expected to be independent of the gas density, even over the range of about 10^{20} to 1 in density, embraced by the range of atmospheres considered.

Actually another factor enters in these galactic nebulae, which changes the nature of the discharge propagation process; however, it does not materially alter the above argument, as will be seen later.

STELLAR POPULATIONS I AND II

Another major question on which the discharge theory would appear to have an important bearing is that of the origin of the two stellar Populations in the galaxies (1q). Globular and elliptical nebulae, which are those in which the main galactic discharge has still to occur (1a), contain stars of Population II. These, on the view now proposed, are the oldest stars which have been formed contemporaneously with the

development of the rotational form of the nebula, and with the building up of the generally radial electric field in the nebula's gaseous atmosphere, which envelops the stars of Population II. The electrical breakdown of this atmospheric electric field results in the development of either an irregular nebula, from a globular nebula, or a more or less well defined spiral nebula, from a more or less markedly elliptical nebula. The older Population II stars in the nebula will be little affected by the occurrence of the discharges. The latter will, however, have a considerable effect on the disposition of gas and dust in the nebula. This will be collected into the discharge channels—the spiral arms—by the magnetic pinch effect, a deduction which has been in fact amply confirmed by various observations, optical and radio. There, in gas of greatly increased density, a second population of younger smaller stars will be formed relatively quickly.

On this view, therefore, this second population of stars, which corresponds to Baade's Population I, should be formed along discharge channels, superposed on, or threading through, the general aggregate of Population II stars, which had been formed earlier in the original globular or elliptical phase of the nebula.

That this conclusion agrees well with observation will be seen from the following description (**24**) of what is actually observed, in which the italics are the writer's: "A spiral galaxy combines the properties of irregular and elliptical nebulae. The flattened spiral arms are populated by the same objects that characterize irregular systems—dust, gas and blue super giants. *The spiral structure is imbedded in, and rotates within, a structureless sub-stratum that resembles an elliptical galaxy in general features and, in particular, in the objects that populate it.*"

A NEW THEORY OF PROPAGATION OF COSMICAL ELECTRICAL DISCHARGES

A main idea behind the present account, as expressed in the introductory section, has been the study of various phenomena—atmospheric electrostatic field-building, electrical discharge characteristics, etc.—on a wide variety of scales. The new theory of discharge propagation now to be considered applies, however, only to the breakdown of electrostatic fields in cosmical atmospheres, and does not apply at all in normal long sparks or terrestrial lightning discharges. For the latter the theory that breakdown to a thermally ionized column of arc discharge is complete during the leader stroke still applies (**1g, h**). The theory now proposed is merely a development of that conception which becomes applicable when the temperature in the leader stroke reaches a sufficiently high value—of the order of 8 million degrees.

The writer has emphasized above, and in a recent note (**1n**), that, so far as the normal process of voltage breakdown is concerned, there is no reason to expect that the velocity of propagation of the breakdown process will vary with gas density. However, in these long cosmical

electrical discharges a point will be reached at which a radical change will occur in the whole propagation process. In the discharge channel already formed a jet of gas will be generated, which will flow along the axis of the channel towards its advancing head. As the temperature of the channel rises, so also will the velocity of this jet. When this velocity reaches about 5×10^7 cm. per second, that is, when the axial gas temperature reaches about 8 million degrees absolute, then the velocity of the jet of hot gas will exceed that of the normal process of voltage breakdown in a hydrogen atmosphere, which is probably less than 5×10^7 cm. per second. Thereafter the propagation will depend on the jet of hot gas, and the velocity of propagation will depend upon its temperature. Velocities of propagation of up to about 4000 km. per second will thus become a possibility.

MAGNETIC STORMS

The last remark in the previous section may help to solve an outstanding difficulty which confronts even the electrical discharge theory of those magnetic storms which are observed to follow events at the sun's surface by periods of 1 to 4 days. Whereas no gas velocities greater than 600 or 700 km. per second have been observed at or near the sun's surface, the shorter of these two periods, 1 day, represents an *average* velocity of the jet of particles causing the magnetic storm of over 2000 km. per second. This situation has been rendered even more perplexing by Meinel's recent observation (**25**), that during aurorae and the accompanying magnetic storms protons enter the Earth's upper atmosphere at velocities of over 3,500 km. per second, or about five times the maximum velocity so far observed in outbursts near the sun's surface.

It will be seen that, applied to the theory (**1a**) that the time interval represents the time required for the propagation of an electrical discharge through a tenuous solar atmosphere, the new developments on discharge propagation offer a possible solution. For, as has already been shown, electrical discharges *can* accelerate particles up to just about the maximum velocities so far observed in these particles comprising magnetic storms.

Indeed the existence of this upper limit of about 3000 or 4000 km. per second to these relative velocities in a wide variety of discharge conditions in cosmical atmospheres suggests that the corresponding discharge temperature, namely about 400 million degrees absolute, is that at which thermonuclear processes become of paramount importance in these cosmical electrical discharges.

EPILOGUE

An attempt has been made to show that a great extension of the field of electrical discharges in gases may result from a reassessment of

many astrophysical phenomena from the point of view outlined in the preceeding pages.

In the letter referred to earlier in this paper, Benjamin Franklin quoted a passage from the "Minutes" he kept of his experiments, in which he had enumerated twelve particulars in which the "electrical fluid agrees with lightning." He continued: "The electric fluid is attracted by points. We do not know whether this property is in lightning. But since they agree in all the particulars wherein we can already compare them, is it not probable that they agree likewise in this? Let the experiment be made." The last sentence is surely one of the most pregnant in the history of electricity, and one wonders if perchance it was known to Marconi! In suggesting a step of still greater ratio in the study of this same field of electricity in gases, the writer cannot unfortunately end this note on the intercomparison of the various fields with a similar suggestion. He can only suggest that the observations made in some branches of the wider field of astrophysics should be studied from the new point of view, and hopes he has demonstrated that the first fruits of so doing are at least promising.

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A SIMILARITY SOLUTION FOR A BLAST IN FREE SPACE*

BY

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ABSTRACT

A similarity solution is given for a spherical blast in free space. The solution given satisfies the equations of gas dynamics, *viz.*: (*a*) conservation of mass; (*b*) conservation of momentum; and (*c*) conservation of energy. In addition, the usual gas laws are assumed to apply, namely: (*d*) $p = \rho RT$ and (*e*) $U = \dfrac{RT}{\gamma - 1} = \dfrac{p}{\rho(\gamma - 1)}$.

The solution obtained in the first part applies exactly to the case of a contained explosion, that is, an explosion such that along the boundary of the sphere (exploding region) an equalizing pressure is at all times maintained which is just equal to the boundary pressure. This condition is one which, physically, can occur if the containment could, for example, be maintained by means of a magnetic or similar field external to the bulk of the gas.

It is then shown that the solution obtained may also be approximately valid without a containing boundary pressure but with a rim (or boundary) shock wave satisfying the ideal gas Rankine-Hugoniot shock conditions, although this is speculative.

DEVELOPMENT OF THE THEORY

An investigation will first be made of a possible solution to the hydrodynamic equations and the applicability of this solution will then be discussed. The analysis will be confined to the case of a spherical blast. (The same procedure can be used for cylindrical blasts and very likely for other curvilinear systems as well, although this last has not been investigated.) Because the blast is symmetric, the governing equations are,

continuity

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial r} + \frac{2\rho u}{r} = 0 \tag{1}$$

momentum,

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} \tag{2}$$

ideal gas laws,

$$\left.\begin{aligned} p &= \rho RT \\ U &= \frac{RT}{\gamma - 1} = \frac{p}{\rho(\gamma - 1)} \end{aligned}\right\} \tag{3}$$

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energy,

$$\frac{D}{Dt}\left(\tfrac{1}{2}u^2 + U\right) + \frac{1}{\rho}\operatorname{div}(p\bar{u}) = 0 \tag{4}$$

where $\frac{D}{Dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial r}$ is the time derivative seen by material which is moving at velocity u. Now introduce the similarity transformation

$$\xi = \frac{r}{t}$$

or

$$\frac{\partial}{\partial t} = -\frac{\xi}{t}\frac{d}{d\xi}$$

$$\frac{\partial}{\partial r} = \frac{1}{t}\frac{d}{d\xi};$$

here, $\frac{d}{d\xi}$ denotes the partial derivation $\left(\frac{\partial}{\partial \xi}\right)_t$, and it has been assumed that all quantities are functions of ξ but not of t, so that $\left(\frac{\partial}{\partial t}\right)_\xi$ may be set equal to zero. This implies that a time dependent phenomenon, which in the physical plane is represented by a different curve for each time, is shown by a single curve in a ξ plane, which holds for all time.

With this transformation, Eqs. 1 and 2 become the following differential equations

$$-\xi\frac{d\rho}{d\xi} + \frac{d(\rho u)}{d\xi} + \frac{2\rho u}{\xi} = 0 \tag{1a}$$

$$-\xi\frac{du}{d\xi} + u\frac{du}{d\xi} = -\frac{1}{\rho}\frac{dp}{d\xi} \tag{2a}$$

and Eqs. 3 and 4 require that everywhere in the ξ-plane

$$\frac{\rho^\gamma}{p} = \text{constant} \tag{3a}$$

implying conservation of entropy in the frame of the moving fluid.

Consider solutions of these equations, for ρ, of the form

$$\rho = A\xi^n. \tag{5}$$

Then, from Eqs. 1*a* and 2*a* it follows that

$$u = B\xi \tag{6}$$

$$p = C\xi^{n+2} \tag{7}$$

are acceptable solutions. Hence, from Eq. 3*a*, there follows the relation

$$\gamma = \frac{n+2}{n}. \tag{8}$$

Furthermore, substituting Eqs. 5 and 6 in 1*a*, one finds

$$B = \frac{n}{n+3} \tag{9}$$

and from 2*a*,

$$C = \frac{3nA}{(n+2)(n+3)^2}. \tag{10}$$

Thus, the values, of ρ, u and p which formally satisfy the gas dynamic equations (in the similarity form assumed herein) are

$$\left.\begin{aligned} \rho &= A\xi^n \\ u &= n(n+3)^{-1}\xi \\ p &= 3n(n+2)^{-1}(n+3)^{-2}A\xi^{n+2} \end{aligned}\right\}. \tag{11}$$

The total mass and energy in the blast will now be determined as follows:

The total mass in the exploding sphere, at a time t_1 and "position" ξ_1, is

$$M = t_1^3 \int_0^{\xi_1} 4\pi\xi^2\rho d\xi \tag{12}$$

or

$$M = \frac{4\pi}{n+3} A\xi_1^{n+3}t_1^3 \tag{12a}$$

and the total energy in the same sphere is, obtaining U from Eq. 3,

$$E = t_1^3 \int_0^{\xi_1} \rho(\tfrac{1}{2}u^2 + U)4\pi\xi^2 d\xi \tag{13}$$

or

$$E = \frac{2\pi n^2}{(n+2)(n+3)^2} A\xi_1^{n+5}t_1^3. \tag{13a}$$

To recapitulate:

A solution has been obtained for a sphere exploding in such a manner that it is similar to itself with time. For this condition, the

phenomenon may be represented in a ξ-plane, with outer boundary ξ_1. The velocity, density and pressure at any point are given by

$$\left.\begin{aligned} u &= n(n+3)^{-1}\xi \\ \rho &= A\xi^n \\ p &= 3n(n+2)^{-1}(n+3)^{-2}A\xi^{n+2} \end{aligned}\right\} \tag{11}$$

and the total energy and total mass in the ball, at any time t_1 are given by

$$E = 2\pi n^2(n+2)^{-1}(n+3)^{-2}A\xi_1^{n+5}t_1^3 \tag{13a}$$

$$M = 4\pi(n+3)^{-1}A\xi_1^{n+3}t_1^3. \tag{12a}$$

PROCEDURE

Because $u = C_1\xi$, where $C_1 = \dfrac{n}{n+3} < 1$ the particles at ξ_1 travel with velocity less than ξ_1. Hence the dimension ξ_1 itself varies with time, and an elementary computation indicates that this variation is as follows:

$$\frac{\Delta\xi_1}{\xi_1} = -\frac{\Delta t_1}{t_1}(1 - C_1) \tag{14}$$

or

$$\xi_2 = \xi_1\left(\frac{t_1}{t_2}\right)^{3/(n+3)} \tag{14a}$$

in which ξ_2 is the rim value at time t_2.

Because of this variation of ξ_1 with time the conservation laws must be rechecked—particularly those of over-all mass and over-all energy.

Thus for mass, using Eqs. 12*a* and 14*a*, one finds

$$\Delta M = 0 \tag{15}$$

which affirms that despite the change in ξ_1, mass is conserved, as required. On the other hand, for energy, making use of Eqs. 13*a* and 14, one obtains

$$\Delta E = -\frac{6}{n+3}2\pi n^2(n+2)^{-1}(n+3)^{-2}A\xi_1^{n+5}t_1^2\Delta t_1 \tag{16}$$

which indicates that energy is *lost* from the system or *stored* in the surroundings.

The physical explanation for ΔE is simple—it is just equal to the work done on a containing pressure equal to the pressure at the rim of the blast. The verification follows:

Work done by p_{ξ_1} = (Force) (Distance)

$$= [3n(n+2)^{-1}(n+3)^{-2}A\xi_1^{n+2}][4\pi\xi_1^2 t_1^2]\left[\frac{n}{n+3}\xi_1\Delta t_1\right]$$

$$= \frac{6}{n+3}2\pi n^2(n+2)^{-1}(n+3)^{-2}A\xi_1^{n+5}t_1^2\Delta t_1 \tag{17}$$

as required.

Incidentally, from Eqs. 12*a*, 13*a*, and 14*a*, one finds

$$E_2 = E_1\left(\frac{t_1}{t_2}\right)^{6/(n+3)} \tag{18}$$

from which the energy remaining in the blast at all times may be determined.

THE RIM SHOCK CONDITION

A solution will now be given using the methods derived in the paper for the field within a particular intense explosion. The solution obtained, in addition to satisfying the usual conservation and gas laws, also satisfies the following two compatibility conditions:

(1) The total energy within the blast remains constant with time

(2) The density at the rim of the blast remains constant with time.

Condition (1) represents the over-all energy constancy assumed and condition (2), which follows from the assumption of a very strong rim shock and the corresponding Rankine-Hugoniot equations, is necessary in order to obtain an approximate agreement with the boundary conditions. The problem in this form is the one considered by Taylor[2] and he found,

Rim radius, r_1, is proportional to $t^{2/5}$

Rim pressure, p_1, is proportional to r_1^{-3}

Rim velocity, u_1, is proportional to $r_1^{-3/2}$.

These results will now be obtained using the similarity coordinate of the present paper

$$\xi = \frac{r}{t}$$

and, in this formulation, an approximate solution may be obtained in closed form.

[2] G. I. TAYLOR, "Formation of a Blast Wave by a Very Intense Explosion, I," *Proc. Roy. Soc.*, Ser. A, Vol. 201, pp. 159–174 (1950).

PROCEDURE

Equations 5, 7 and 13 give

$$\rho = A\xi^n \tag{5}$$

$$p = C\rho\xi^2 \tag{7}$$

$$E = Dt^3\rho_1\xi_1^5 \tag{13}$$

where A, C and D are constants, and subscript 1 refers to rim values.

Now, if E is to be a constant (that is, independent of time) while ρ_1 is also to be essentially a constant, it is necessary that (see Eq. 13)

$$t^3\frac{r_1^5}{t^5} = \text{constant} \tag{19}$$

or it is necessary that

$$r_1 \sim t^{2/5}. \tag{20}$$

Also, if ρ_1 is to be essentially constant, then it is necessary (see Eq. 7), since

$$p_1 = B\rho_1\xi_1^2 \tag{21}$$

that

$$p_1 \sim \frac{r_1^2}{t^2} \tag{22}$$

or

$$p_1 \sim \frac{1}{r_1^3}. \tag{23}$$

It was also shown previously (Eq. 6) that $u = B\xi$. Hence

$$u_1 \sim \frac{r_1}{t} \tag{24}$$

or

$$u_1 \sim r_1^{-3/2}. \tag{25}$$

The results given above apply to an intense explosion. It is interesting to note that the density discontinuity across a very strong shock is given by

$$\frac{\rho_1}{\rho_0} = \frac{\gamma + 1}{\gamma - 1} \tag{26}$$

and for the free space blast this requires

$$\gamma \to 1 \tag{27}$$

or, from Eq. 8,

$$n \to \infty. \tag{28}$$

For these values the constant energy free-space blast solution would be attainable with a rim-shock. However, it is very likely that the gas-dynamic, perfect gas laws and Rankine-Hugoniot relations are not applicable in the shock zone for this case and it is doubtful that the simplified shock-front solution applies. It is conceivable, though, that a combined free-space and shock-front solution is approximately applicable.

CONCLUSION

A similarity solution is given for a spherical blast in free space.

The solution satisfies the gas laws and conservation laws and applies exactly, in the form first given, to the case of a contained blast in which pressure equality is maintained along the rim.

Finally, it is shown that a pressure-free, intense shock-wave rim condition may be possible, although conditions at the rim for this case are uncertain and not completely known.

TOPOLOGY AND THE SOLUTION OF LINEAR SYSTEMS*

BY

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ABSTRACT

The concepts of linear graph theory are applied to the study of feedback systems. Two topological matrices, called the exit and entrance matrices, are defined and the transmission matrix of the system is expressed in terms of these matrices. The properties of these matrices are examined, and a relation between nonsingular submatrices and nontouching feedback loops is established. Graph theory and the theory of determinants allow a rigorous proof of Mason's general gain formula. A systematic method, based on the topological formulas derived in the paper, of finding all forward paths and feedback loops without drawing the graph of the feedback system is demonstrated.

INTRODUCTION

A signal flow graph (1, 2)[2] is a convenient picture of a set of simultaneous linear equations. For example, the graph of Fig. 1 represents the set of equations

$$\begin{aligned} x_1 - fx_2 - ex_3 &= y_1 \\ -ax_1 + x_2 - dx_4 &= 0 \\ -bx_2 + x_3 &= 0 \\ -cx_3 + x_4 &= 0. \end{aligned}$$

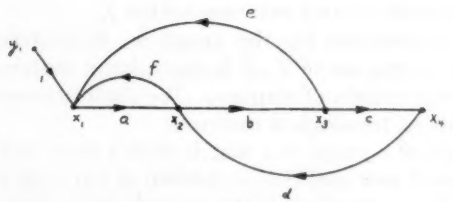

FIG. 1. A signal flow graph.

FIG. 2. Removal of self-loops.

Any independent set of linear equations can be described by a signal flow graph. The equations may be written in the above form merely by dividing each equation by a factor which makes the main diagonal of the matrix of the system contain only 1's. Mason (1, 2) has shown that the solution of the equations depends on the topological properties of the graph, specifically on the forward paths and feedback loops.

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

Mason's results suggest a study of the abstract properties of oriented graphs. In this paper, the concepts of linear graph theory (**3, 4, 5**) are applied to the signal flow graph. Mason's results are rederived from this new point of view. A systematic approach to the mechanical solution of the graph is demonstrated.

Let us assume first that no self-loops are present in the graph. This involves no loss of generality, since self-loops may always be removed by the technique shown in Fig. 2. Also, let us remove all sources from the graph.

Consider a signal flow graph with e *edges* (directed branches) and v *vertices* (nodes). For the graph of Fig. 1, $e = 6$ and $v = 4$. The six edges are a, b, c, d, e, f and the four vertices are x_1, x_2, x_3 and x_4. The graph is completely described by the *transmission matrix* T. The element τ_{ij} of T is the negative of the transmission from vertex i to vertex j, if $i \neq j$, and is 1 if $i = j$. For the graph of Fig. 1,

$$T = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & -a & 0 & 0 \\ -f & 1 & -b & 0 \\ -e & 0 & 1 & -c \\ 0 & -d & 0 & 1 \end{bmatrix} \end{array}$$

Thus

$$\begin{aligned} \tau_{ij} &= -t_{ij}, & i \neq j \\ \tau_{ij} &= 1, & i = j \end{aligned}$$

where t_{ij} is the transmission from vertex i to vertex j, that is, the sum of the weights of all edges leaving vertex i and entering vertex j.

The system of equations represented by the graph is, in matrix form, $T'X = Y$, where T' is the transpose of T, X is the column matrix of unknowns, and Y is the column matrix of sources. We shall express the transmission matrix in terms of topological matrices.

Definition 1. The *exit* matrix of a graph is a matrix with v rows and e columns. The element in the i^{th} row and the j^{th} column is 1 if edge j is incident at vertex i, with the orientation of the arrow *away* from vertex i, and is 0 otherwise. We use the symbol C for the exit matrix.

Definition 2. The *entrance* matrix of a graph is a matrix with v rows and e columns. The element in the i^{th} row and the j^{th} column is 1 if edge j is incident at vertex i, with the orientation of the arrow *toward* vertex i, and is 0 otherwise. We use the symbol D for the entrance matrix. For the graph of Fig. 1,

$$C = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array} \qquad D = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

The rows and columns of C and D are always assumed to be ordered in the same way. Notice that $(C - D)$ is the well-known *incidence* matrix of linear graph theory (4).

Definition 3. The *edge* matrix F is a diagonal matrix of order e. The element in the i^{th} row is the i^{th} edge of the graph, where the ordering of the edges corresponds to the ordering of the columns C and D. For the graph of Fig. 1

$$F = \begin{bmatrix} a & & & & & \\ & b & & & \bigcirc & \\ & & c & & & \\ & & & d & & \\ & \bigcirc & & & e & \\ & & & & & f \end{bmatrix}$$

Theorem 1. $U - CFD' = T$, where the unit matrix is of order v.

Proof: Let $C = [c_{ij}]$, $F = [f_{ij}]$, $D = [d_{ij}]$.
The element q_{ij} in row i and column j of $U - CFD'$ is given by

$$q_{ij} = \delta_{ij} - \sum_{k=1}^{e} \sum_{l=1}^{e} c_{ik} f_{kl} d_{jl}$$

where δ_{ij} is the "Kronecker delta." Since F is a diagonal matrix, the above expression reduces to

$$q_{ij} = \delta_{ij} - \sum_{k=1}^{e} c_{ik} f_{kk} d_{jk}.$$

The quantities c_{ik} and d_{jk} will both be non-zero if and only if edge k leaves vertex i and enters vertex j. Hence

$$q_{ij} = \delta_{ij} - t_{ij}$$

where t_{ij} is the transmission from vertex i to vertex j. Since the element in row i and column j of T is also $\delta_{ij} - t_{ij}$, the theorem follows.

Theorem 2. C and D have exactly one non-zero element per column.

Proof: Each edge leaves exactly one vertex and enters exactly one vertex. Hence the theorem.

Definition 4. An *exit branch* of a vertex is an edge (branch) leaving that vertex. An *entrance branch* of a vertex is an edge entering that vertex. (Compare definitions 1 and 2.)

Theorem 3. The rank of C is n_1, where n_1 is the number of vertices having at least one exit branch. The rank of D is n_2 where n_2 is the number of vertices having at least one entrance branch.

Proof: We shall restrict our attention to C. The proof for D is very similar. Let the vertices which have exit branches be ordered $1, 2, \cdots, n_1$. Let e_i be any exit branch of vertex i, and consider the square submatrix N_1 of C whose columns are $e_1, e_2, \cdots, e_{n_1}$, and whose rows are $1, 2, \cdots, n_1$. Each column of this submatrix has exactly one non-zero element, and no row has more than one non-zero element, for otherwise a vertex would have two exit branches, contradicting the method of formation of N_1. N_1 is clearly a unit matrix, and the rank of C is therefore at least n_1. Only vertices $1, 2, \cdots, n_1$ have exit branches. Therefore all other rows of C contain only 0's. Thus the rank of C is exactly n_1. If the graph contains no sinks, the rank of C is equal to v, the number of vertices in the graph. If all sources are removed from the graph, the rank of D is also equal to v.

Theorem 4. Let a square submatrix S of C (or D) be defined by edges $e_1, e_2, \cdots, e_k$ and vertices $1, 2, \cdots, k$. Then S is nonsingular if and only if each vertex of the set $(1, 2, \cdots, k)$ has exactly one exit (or entrance) branch from the set $(e_1, e_2, \cdots, e_k)$.

Proof: We consider the matrix C. The proof for D is identical. First suppose that each vertex of the set $(1, 2, \cdots, k)$ has exactly one exit branch from the set $(e_1, e_2, \cdots, e_k)$. Consider the reduced graph formed by edges $e_1, e_2, \cdots, e_k$. S is the exit matrix of this graph. By theorem 3, the rank of S is equal to k, since k is the number of vertices which have exit branches. Thus the rank of S is equal to its order, and S is nonsingular.

Now suppose S is nonsingular. If vertex i $(1 \leq i \leq k)$ has no exit branches from the set $(e_1, e_2, \cdots, e_k)$ the row i of S consists entirely of zeros, contradicting the hypothesis of nonsingularity. If vertex i has two exit branches e_m and e_n $(1 \leq m < n \leq k)$ then columns m and n are identical, again contradicting the nonsingularity of S. Hence the theorem.

Theorem 5. Corresponding submatrices of C and D (submatrices defined by the same rows and columns) are *both* nonsingular if and only if the edges corresponding to the columns of the submatrices form a single feedback loop or a set of nontouching feedback loops. (A set of nontouching feedback loops is a set of loops no two of which have a vertex in common.)

Proof: From theorem 4, for corresponding submatrices to be nonsingular, it is necessary and sufficient that the edges corresponding to the columns of the submatrices form a subgraph with exactly one edge leaving each vertex and exactly one edge entering each vertex. Each vertex of the subgraph is therefore of degree two, that is, there are two edges incident, one entering and one leaving. Thus the columns of the

submatrices correspond to a feedback loop or a union of feedback loops. Since the degree of each vertex is 2, no two feedback loops can have a vertex in common, since this would make the degree of the common vertex greater than two.

In the graph of Fig. 1, edges a, b and e form a feedback loop. The corresponding nonsingular submatrices S_1 and S_2 of C and D, respectively, are:

$$S_1 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} a & b & e \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array} \qquad S_2 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} a & b & e \end{array} \\ \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

Theorem 6. All nonsingular submatrices of C and D have determinant ± 1.

Proof: Since all columns of nonsingular submatrices have exactly one non-zero element, the determinant of each submatrix has only one term. Since all elements of the submatrix are either 1 or 0, the theorem follows.

Theorem 7. Suppose corresponding submatrices in C and D are nonsingular. Let $L_1, L_2, \cdots, L_r$ be the nontouching feedback loops associated with the columns of the submatrices. The two corresponding submatrices have the same determinant if and only if there is an even number of loops in the set $\{L_1, L_2, \cdots, L_r\}$ having an even number of edges. The number of loops having an odd number of edges is immaterial.

Proof: Consider the feedback loop L_i. Suppose that the edges of L_i are $e_1, e_2, \cdots, e_k$ and the vertices of L_i are $1, 2, \cdots, k$. In addition, suppose that edge e_j leaves vertex j and enters vertex $j+1$, for $j = 1, 2, \cdots, k-1$, with e_k leaving vertex k and entering vertex 1. The submatrix of C corresponding to L_i is

$$S_{1i} = \begin{array}{c} \\ 1 \\ 2 \\ \vdots \\ k \end{array} \begin{array}{c} \begin{array}{cccc} e_1 & e_2 & \cdots & e_k \end{array} \\ \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ & & & \\ 0 & 0 & \cdots & 1 \end{bmatrix} \end{array}$$

The submatrix of D corresponding to L_i is

$$S_{2i} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ \vdots \\ k \end{array} \begin{array}{c} \begin{array}{cccc} e_1 & e_2 & & e_k \end{array} \\ \begin{bmatrix} 0 & 0 & \cdots & 1 \\ 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ & & & \\ 0 & 0 & \cdots & 0 \end{bmatrix} \end{array}$$

S_{1i} and S_{2i} are of course nonsingular. S_{2i} may be made to coincide with S_{1i} by $k-1$ interchanges of adjacent rows. Row 1 of S_{2i} must be moved to the bottom, while the relative position of all other rows remains unchanged. Thus, if k is odd, $|S_{1i}| = |S_{2i}|$ and if k is even, $|S_{1i}| = -|S_{2i}|$.

If S_1 and S_2 are the submatrices of C and D, respectively, whose columns correspond to all the edges of the set $\{L_1, \cdots, L_r\}$, then

$$|S_1| = |S_{11}||S_{12}| \cdots |S_{1r}|$$
$$|S_2| = |S_{21}||S_{22}| \cdots |S_{2r}|$$

This follows from the fact that no two feedback loops in the set $\{L_1, \cdots, L_r\}$ have a vertex in common. $|S_{1i}| = -|S_{2i}|$ if and only if L_i has an even number of edges. Hence $|S_1| = |S_2|$ if and only there is an even number of loops with an even number of edges.

We are now ready to establish Mason's general gain formula.

Definition 5. The *weight* of a feedback loop is the product of the weights of the edges of the loop.

Theorem 8. (Mason) $\Delta = |T| = 1$ minus the sum of the weights of all feedback loops, plus the sum of the products of the weights of all nontouching feedback loops taken two at a time, minus the sum of the products of the weights of all nontouching feedback loops taken three at a time, plus $\cdots + (-1)^m$ (the sum of the products of the weights of all nontouching feedback loops taken m at a time). The number m is the maximum number of non-touching feedback loops in the graph. For the graph of Fig. 3, $m = 2$, and

$$\Delta = 1 - ad - be - cf - abg + adcf.$$

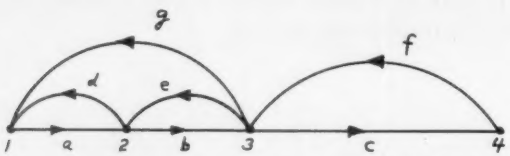


FIG. 3. A signal flow graph with two nontouching feedback loops.

Proof: Let $R = CFD'$. Then $T = U - R$, by theorem 1. Expand $|T|$ by diagonal elements (6, p. 87).

$$\Delta = |T| = 1 - r_{11} - r_{22} - \cdots - r_{vv} + |r_{11}r_{22}| + |r_{11}r_{33}| + \cdots + |r_{v-1,v-1}r_{vv}| - |r_{11}r_{22}r_{33}| - |r_{11}r_{22}r_{44}| - \cdots - |r_{v-2,v-2}r_{v-1,v-1}r_{vv}| + \cdots + (-1)^v|R|.$$

The term $|r_{11}r_{22}\cdots r_{kk}|$ is the principal minor of R formed by rows 1, 2, $\cdots$, k and columns 1, 2, $\cdots$, k. By theorem 1, the elements

along the main diagonal of R are 0, and thus $r_{11} = r_{22} = \cdots = r_{vv} = 0$.

Consider a typical principal minor $|r_{11}r_{22}\cdots r_{kk}|$. In order to evaluate this minor, we delete rows $k+1, k+2, \cdots, v$ from R, and also delete like-numbered columns. This is equivalent to deleting rows $k+1, k+2, \cdots, v$ from both C and D. Since R is the product of the two matrices CF and D', we may find all principal minors of R by using the Binet-Cauchy theorem (6, pp. 85–86). A typical term in $|r_{11}r_{22}\cdots r_{kk}|$ is $\pm w_a w_b \cdots w_z$, where w_i is the weight of edge e_i, and edges $e_a, e_b, \cdots, e_z$ correspond to the columns of nonsingular submatrices of C and D whose rows are $1, 2, \cdots, k$. The sign will be positive if the corresponding submatrices have the same determinant, negative otherwise. By theorem 5, all single feedback loops and all nontouching sets of feedback loops will appear, provided that the edges of these loops are incident only at vertices $1, 2, \cdots,$ or k. Since all possible combinations of vertices, taken $1, 2, \cdots, v$ at a time, are present in the expansion, all possible single feedback loops and all possible sets of nontouching feedback loops will appear. It remains to show that the sign of each term is correct. Let $t = \pm w_a w_b \cdots w_e$ be a term in the expansion of $|r_{11}\cdots r_{kk}|$, and suppose this term corresponds to the feedback loops $L_1, L_2, \cdots, L_r$, and to the vertices $1, 2, \cdots, k$. If there is an even number of loops having an even number of edges, the sign of t in $|r_{11}r_{22}\cdots r_{kk}|$ will be positive by theorem 7. If there is an even number of loops having an odd number of edges, the total number of loops is even, and the total number of edges in the set $\{L_1, L_2, \cdots, L_r\}$, namely k, is also even. The latter statement implies that $|r_{11}r_{22}\cdots r_{kk}|$ is prefixed with a positive sign. Thus t will have a positive sign in the expansion of $|T|$. Since there is an even number of loops, t is the product of nontouching feedback loops taken $2n$ at a time, for some non-negative integer n. Therefore the positive sign of t is consistent with the statement of the theorem. Similar reasoning shows that all terms in $|T|$ have the proper sign. The steps in this reasoning are summarized in tabular form below.

No. of loops in $\{L_1, \cdots, L_r\}$ with an even no. of edges	even	even	odd	odd
No. of loops in $\{L_1, \cdots, L_r\}$ with an odd no. of edges	even	odd	even	odd
Total number of loops	even	odd	odd	even
Total number of edges	even	odd	even	odd
Sign of t in $\lvert r_{11}\cdots r_{kk}\rvert$	+	+	−	−
Sign of $\lvert r_{11}\cdots r_{kk}\rvert$	+	−	+	−
Sign of t	+	−	−	+

Thus, the theorem follows.

Now let us turn our attention to the solution of the set of linear equations described by the graph. Suppose we wish to find the transmission from a source y_1 to a sink x_v. By Cramer's rule (6, p. 87), $x_v/y_1 = T_{v1}/|T|$, where T_{v1} is the cofactor of the element $\tau_{v1} = -t_{v1}$ of T. T_{v1} may be written as:

$$T_{v1} = (-1)^{v+1} \begin{vmatrix} -t_{12} & -t_{13} & \cdots & -t_{1v} \\ & Q & & -t_{2v} \\ & & & \vdots \\ & & & -t_{v-1,v} \end{vmatrix}$$

where

$$Q = \begin{array}{c} \\ 2 \\ 3 \\ \cdot \\ \cdot \\ \cdot \\ v-1 \end{array} \begin{array}{c} \begin{array}{cccc} 2 & 3 & \cdots & v-1 \end{array} \\ \begin{bmatrix} 1 & -t_{23} & \cdots & -t_{2,v-1} \\ -t_{32} & 1 & & -t_{3,v-1} \\ \cdot & \cdot & & \\ \cdot & \cdot & & \\ \cdot & \cdot & & \\ -t_{v-1,2} & -t_{v-1,3} & \cdots & 1 \end{bmatrix} \end{array}$$

By the Cauchy expansion of a bordered determinant (6, pp. 74–75),

$$T_{v1} = (-1)^{v+1}\left[(-t_{1v})(-1)^{v}|Q| + \sum_{i,j=2}^{v-1} t_{iv}t_{1j}T_{v1,1v,ij}\right]$$

$T_{v1,1v,ij}$ is $(-1)^{1+v+i+j}$ times the determinant formed from T_{v1} by deleting rows i and 1, and columns v and j. Clearly

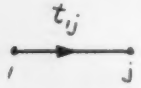
$$T_{v1,1v,ij} = (-1)^{1+v+i+j}|Q_{ij}|$$

where $|Q_{ij}|$ is the minor formed from Q by deleting row i and column j. Thus

$$T_{v1} = t_{1v}|Q| + \sum_{i,j=2}^{v-1} t_{iv}t_{1j}Q_{ij}$$

where Q_{ij} is the cofactor of the element q_{ij} of Q, with $i, j = 2, 3, \cdots, v-1$.

The graph G' corresponding to Q is the original graph G corresponding to T with vertices 1 and v removed, as well as all edges incident at these vertices (see Fig. 4).



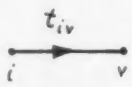
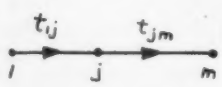


FIG. 4. G' does not contain t_{1j} or t_{iv}.



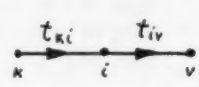


FIG. 5. G'' does not contain t_{1j}, t_{jm}, t_{ki}, or t_{iv}.

Q is the transmission matrix of G', and the cofactor Q_{ij} is of interest in evaluating the transmission from vertex j to vertex i. By the same reasoning as before,

$$Q_{ij} = t_{ji}|P| + \sum_{\substack{k,\,m=2 \\ k,\,m \neq i \text{ or } j}}^{v-1} t_{ki}t_{jm}P_{km}$$

where the graph G'' corresponding to P is the graph G' with vertices i and j removed, as well as all edges incident at these vertices (see Fig. 5).

The process described above may be continued until a sequence $t_{1i}t_{jm}t_{mb}\cdots t_{pk}t_{ki}t_{iv}$ is obtained which forms an oriented (forward) path between vertices 1 and v. The determinant which multiplies this forward path will be the transmission determinant of the graph formed from the original graph by deleting the forward path, its vertices, and all edges incident at these vertices. Each of these transmission determinants ($|Q|$, $|P|$, etc.) is of order $v - 2n$, for some non-negative integer n. We may now state the following theorem.

Theorem 9. (Mason) $T_{v1} = \sum_k P_k\Delta_k$, where P_k is the product of the weights of edges of a forward path between vertices 1 and v, and Δ_k is the transmission determinant of the graph formed from the original graph by deleting the forward path, its vertices, and all edges incident at these vertices. From theorems 8 and 9, we may now write the expression for the graph gain as follows:

$$G = \frac{x_v}{y_1} = \frac{T_{v1}}{|T|} = \frac{\sum_k P_k\Delta_k}{\Delta}.$$

APPLICATIONS

From the point of view of machine computation, it is desirable to have a systematic method of evaluating the transmission determinant and its various cofactors. Inspection of the graph, while convenient for manual calculation when the order of the system is not too high, becomes inefficient when the graph contains a large number of forward paths and feedback loops. The following procedure is perhaps suitable for the analysis of linear systems by a digital computer.

Step 1

Given the set of simultaneous equations to be solved, construct the matrices C and D without drawing the graph. The method of doing this is illustrated for the equations associated with the graph of Fig. 1. These equations are repeated for convenience.

$$\begin{aligned} x_1 - fx_2 - ex_3 &= y_1 \\ -ax_1 + x_2 - dx_4 &= 0 \\ -bx_2 + x_3 &= 0 \\ -cx_3 + x_4 &= 0 \end{aligned}$$

The letter a multiplies x_1, and therefore there will be a 1 in column a, row 1, of C. Similarly, there will be a 1 in column b, row 2; column c, row 3; column d, row 4; column e, row 3; and column f, row 2. All other elements of C will be 0. The letters e and f occur in the first row, and therefore there will be a 1 in columns e and f, row 1, of D. Similarly, there will be a 1 in columns a and d, row 2; column b, row 3; and column c, row 4. All other elements of D will be 0.

Step 2

Systematically examine all corresponding submatrices of C and D. If pairs of corresponding submatrices are both nonsingular, form the term $w_a w_b \cdots w_c$ where $e_a, e_b, \cdots, e_c$ are the edges corresponding to the columns of the submatrices. The sign of the term is determined from the following table.

Determinants of Submatrices	No. of Columns in Submatrices	Sign of $w_a w_b \cdots w_c$
Same	even	+
Same	odd	−
Different	even	−
Different	odd	+

Addition of all such terms gives the transmission determinant.

Step 3

Assume that a source is connected to vertex 1, and the signal at vertex v is desired. In other words, we wish to find x_v/y_1. Find all forward paths between vertices 1 and v by the following procedure. Select any edge leaving vertex 1. This can be done by inspection of C. Find out, by observation of D, where this edge terminates. For the purpose of this discussion, let the terminal vertex be called vertex 2. If vertex 2 is vertex v, we have a forward path. If not, select an edge leaving vertex 2. If this edge terminates on vertex 1, stop and select another edge. If this edge terminates on a new vertex, say vertex 3, proceed as before. In this way, all possible forward paths are found.

Step 4

Suppose $e_1 e_2 \cdots e_k$ is a forward path, touching vertices 1, 2, $\cdots$, v. Delete columns $e_1, e_2, \cdots, e_k$ and rows 1, 2, $\cdots$, v from C and D. Re-

peat steps 1 and 2 for the reduced matrices. In this way, all products $P_k\Delta_k$ are found.

It has been brought to the attention of the author that a topological analysis of a linear graph corresponding to a set of simultaneous linear equations has been carried out independently by C. L. Coates in the *IRE Transactions on Circuit Theory*, June 1959.

The construction of the "flowgraph" of Coates is somewhat different from that of the "signal flow graph" of Mason. Coates derives a general gain formula which involves roughly the same amount of computational labor as Mason's formula.

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KINEMATIC PROPERTIES OF THE THREE-GEAR DRIVE

BY

M. F. SPOTTS[1]

ABSTRACT

When an indexing or intermittent motion mechanism is required, the three-gear drive has a number of advantages such as smooth motion, simplicity, and low cost. However, in the past, difficulties of making precise calculations have undoubtedly restricted a wider application for this device. The equations derived in this article will permit an accurate determination to be made of the rotation, angular velocity, and acceleration of the output shaft. Two worked-out numerical examples are included.

In the field of automatic machinery, it is frequently necessary for the output shaft to have an intermittent motion, while the input shaft is turning uniformly. It is usually desirable that the curves representing the velocity and acceleration of the output shaft be smooth and free from discontinuities. The masses are then accelerated and decelerated gradually, and the mechanism is free from shock and impact.

The three-gear drive illustrated in Fig. 1 can be arranged to serve such a purpose. By suitable choice of dimensions, the output gear 3 can be given a short dwell period or even a reversal of motion during part of the cycle. Ordinary spur gears can be used if desired. Gear 1 is mounted off center at O_{ad}. The gears are held in mesh by links AB and BO_{cd}. The arrangement of parts thus gives a four-bar linkage with lengths of links a, b, c, and d. Let it first be assumed that these lengths are known. Link a or gear 1, is the driver which is turning at a constant angular velocity of ω_1 rad/sec. Equations will now be derived which will give the rotation ϵ of driven gear 3 for an angular rotation γ_1 of the driving link.

When links a and b are in a straight line as shown in Fig. 1a, the angles of the resulting triangle are β, γ, and δ. Let

$$s = \tfrac{1}{2}(a + b + c + d). \tag{1}$$

By trigonometry[2]

$$\cos^2 \frac{\delta}{2} = \frac{s(s-d)}{(a+b)c} \tag{2}$$

$$\cos \delta = 2 \cos^2 \frac{\delta}{2} - 1 = \frac{2s(s-d)}{(a+b)c} - 1. \tag{3}$$

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[2] See O. W. ESHBACH, "Handbook of Engineering Fundamentals," New York, John Wiley & Sons, Inc., second edition, 1952, pp. 2–74.

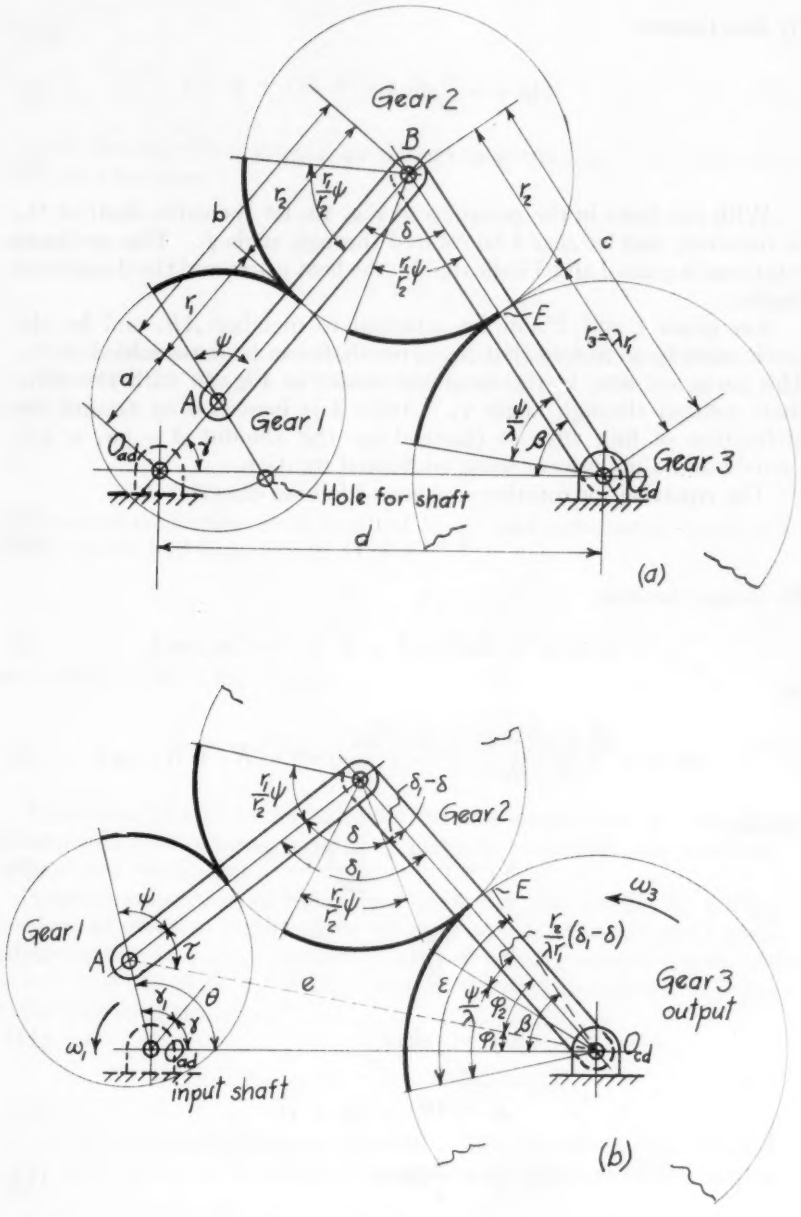


FIG. 1. The three-gear drive.

By sine theorem

$$\sin \gamma = \frac{c}{d} \sin \delta \tag{4}$$

$$\beta = 180° - (\gamma + \delta). \tag{5}$$

With the links in the positions of Fig. 1*a*, let the drive shaft at O_{ad} be removed, and let gear 1 be rotated through angle ψ. This produces rotations in gears 2 and 3 indicated by the heavy arcs and the designated angles.

Let gears 1 and 2 now be attached to member AB, and let the mechanism be so moved that the drive shaft can be reassembled at O_{ad}. The parts are now in the locations shown in Fig. 1*b* with the drive shaft rotated through angle γ_1. Angle δ is increased to δ_1, and the inclination of link BO_{cd} is changed by the amount $\beta - (\varphi_1 + \varphi_2)$. Gears 2 and 3 underwent some additional rotation.

The equation for rotation ϵ of gear 3 follows directly.

$$\theta = \gamma + \gamma_1 \tag{6}$$

By cosine theorem,

$$e^2 = a^2 + d^2 - 2ad \cos \theta = b^2 + c^2 - 2bc \cos \delta_1 \tag{7}$$

or

$$\cos \delta_1 = \frac{b^2 + c^2 - a^2 - d^2}{2bc} + \frac{ad}{bc} \cos \theta = K_1 + K_2 \cos \theta \tag{8}$$

where

$$K_1 = \frac{b^2 + c^2 - a^2 - d^2}{2bc} \tag{9}$$

$$K_2 = \frac{ad}{bc} \tag{10}$$

$$\sin \tau = \frac{c}{e} \sin \delta_1 \tag{11}$$

$$\varphi_2 = 180 - (\delta_1 + \tau) \tag{12}$$

$$\sin \varphi_1 = \frac{a}{e} \sin \theta \tag{13}$$

$$\psi = \theta + \varphi_1 - \tau. \tag{14}$$

Then

$$\epsilon = \beta - (\varphi_1 + \varphi_2) + \frac{r_2}{\lambda r_1}(\delta_1 - \delta) + \frac{\psi}{\lambda}. \tag{15}$$

For the case where gears 1 and 3 have the same size, $\lambda = 1$, the equation for ϵ becomes

$$\epsilon = \theta - \gamma - C\delta + C\delta_1 \tag{16}$$

where

$$C = 1 + \frac{r_2}{r_1}. \tag{17}$$

Values of ϵ can be computed and plotted from these equations.

Equation 16 can be differentiated to find the velocity ω_3 of the output shaft

$$\frac{d\epsilon}{dt} = \frac{d\theta}{dt} + C\frac{d\delta_1}{dt}.$$

Differential coefficient $d\theta/dt$ is equal to ω_1, and differential coefficient $d\delta_1/dt$ can be had from Eq. 8. Then

$$\omega_3 = \omega_1\left[1 + \frac{K_2 C \sin\theta}{\sin\delta_1}\right] \tag{18}$$

and substitution of Eq. 8 gives

$$\omega_3 = \omega_1\left[1 + \frac{K_2 C\sqrt{1 - \cos^2\theta}}{\sqrt{1 - (K_1 + K_2\cos\theta)^2}}\right]. \tag{19}$$

Equations 18 and 19 are valid for the case where $r_1 = r_3$. Further differentiation to obtain acceleration gives an expression too unwieldy for practical application.

Angular velocity ω_3 of the output shaft can also be had by finding the slope at different points along the ϵ-curve. It can be shown,[3] when the ordinates y_0, y_1, y_2, y_3, and y_4 at five equally spaced points are known, that the slope y_2' of the curve at the middle ordinate is given by the following equation.

$$y_2' = \frac{1}{12h}(y_0 - 8y_1 + 8y_3 - y_4) \tag{20}$$

where h is the spacing along the abscissa. The process can be repeated on the velocity curve to obtain the angular acceleration of the output shaft.

[3] See W. E. MILNE, "Numerical Calculus," Princeton, Princeton University Press, 1949, p. 97.

Example 1

A three-gear drive has $\lambda = 1$, $a = 0.8$ in., $r_1 = 1.0$ in., $r_2 = 1.5$ in., $r_3 = 1.0$ in., and $d = 3.1$ in. For the input shaft, $\omega_1 = 1.875$ rad/sec. Plot curves for displacement, velocity and acceleration of the output shaft at 30-deg. intervals.

Solution

$$b = r_1 + r_2 = 2.5 \text{ in.}$$
$$c = r_2 + r_3 = 2.5 \text{ in.}$$

By Eq. 1, $2s = 0.8 + 2.5 + 2.5 + 3.1 = 8.9$ in. $s = 4.45$ in.

By Eq. 3, $\cos\delta = \dfrac{8.9 \times 1.35}{3.3 \times 2.5} - 1 = 0.45636$ $\delta = 62.848°$

By Eq. 4, $\sin\gamma = \dfrac{2.5}{3.1} \times 0.88980 = 0.71758$ $\gamma = 45.855°$

By Eq. 5, $\beta = 180° - (62.848 + 45.855) = 71.297°$

By Eq. 9, $K_1 = \dfrac{b^2 + c^2 - d^2 - a^2}{2bc} = \dfrac{6.25 + 6.25 - 9.61 - 0.64}{2 \times 2.5 \times 2.5}$
$= 0.1800$

By Eq. 10, $K_2 = \dfrac{ad}{bc} = \dfrac{0.8 \times 3.1}{2.5 \times 2.5} = 0.3968$

By Eq. 8, $\cos\delta_1 = 0.1800 + 0.3968\cos\theta$

By Eq. 16, $\epsilon = \theta - 45.855° - 2.5 \times 62.848° + 2.5\delta_1$
$= \theta - 202.975° + 2.5\delta_1$

Values of ϵ, for assumed values of γ_1, have been computed in Table I. The 30-deg. or $\dfrac{\pi}{6}$-spacing means that h has the following value on the time scale:

$$h = (\pi/6)/1.875 = 0.27925 \text{ sec.}$$

Angular velocity ω_2 for a typical point, as for example γ_1 equal to 90°, is computed as follows by Eq. 20:

$$\omega_2 = \frac{1.0067 - 8 \times 2.0528 + 8 \times 3.7947 - 4.3105}{12 \times 0.27925} = 3.1725 \text{ rad/sec.}$$

TABLE I.—*Computations for Three-Gear Drive of Example 1.*

γ_1	$\theta=\gamma_1+\gamma$	$\cos\theta$	$K_3\cos\theta$	$\cos\delta_1$	δ_1 deg.	$\left(1+\frac{r_2}{r_1}\right)\delta_1$	deg.	ϕ rad.	ω_2 rad/sec	α_2 rad/sec^2
0°	45.86	0.69648	0.27636	0.45636	62.85	157.12	0	0	3.3656	2.3478
30	75.86	0.24438	0.09697	0.27697	73.92	184.80	57.68	1.0067	3.7525	0.5163
60	105.86	−0.27320	−0.10841	0.07159	85.90	214.74	117.62	2.0528	3.6688	−1.0771
90	135.86	−0.71758	−0.28474	−0.10474	96.01	240.03	172.91	3.0178	3.1725	−2.4693
120	165.86	−0.96968	−0.38477	−0.20477	101.82	254.54	217.42	3.7947	2.3354	−3.3918
150	195.86	−0.96196	−0.38171	−0.20171	101.64	254.09	246.97	4.3105	1.3605	−3.3618
180	225.86	−0.69648	−0.27636	−0.09636	95.53	238.82	261.70	4.5676	0.5388	−2.3979
210	255.86	−0.24438	−0.09697	0.08303	85.24	213.09	265.97	4.6421	0.0649	−0.9914
240	285.86	0.27320	0.10841	0.28841	73.24	183.09	265.97	4.6421	0.0062	0.6154
270	315.86	0.71758	0.28474	0.46474	62.31	155.77	268.65	4.6888	0.4222	2.4526
300	345.86	0.96968	0.38477	0.56477	55.61	139.04	281.92	4.9203	1.3351	3.9038
330	375.86	0.96196	0.38171	0.56171	55.83	139.56	312.44	5.4532	2.4774	3.8536
360	405.86	0.69648	0.27636	0.45636	62.85	157.12	360	6.2832	3.3656	2.3478

Computations for angular acceleration α_3 are made in a similar manner. Results are given in the Table, and are plotted in Fig. 2. It should be noted, although the output shaft comes to a stop and reverses slightly, that the graph for acceleration is a smooth and continuous curve.

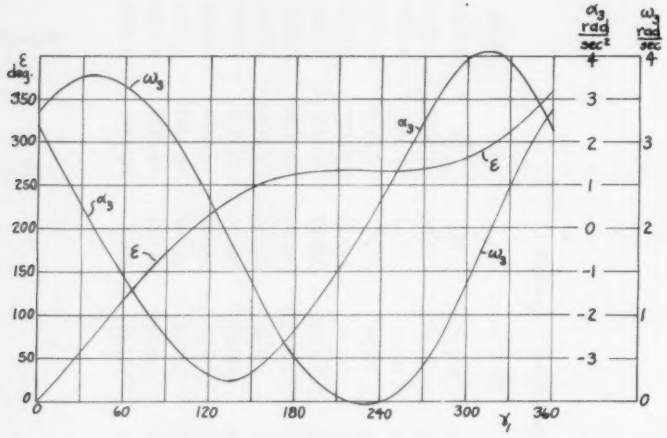


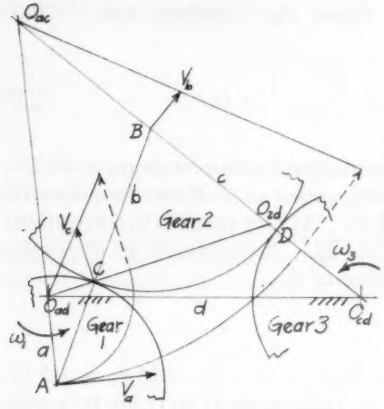
FIG. 2. Angular displacement ϵ, angular velocity ω_3, and angular acceleration α_3 of three-gear drive for $a = 0.8$, $r_1 = 1$, $r_2 = 1.5$, $r_3 = 1$, $d = 3.1$, $\omega_1 = 1.875$ rad/sec.

Equation 20 is convenient for use with a desk calculator since the entire computation can be made as a single operation on the keyboard. Results are valid except for the rare case where extreme precision is required.

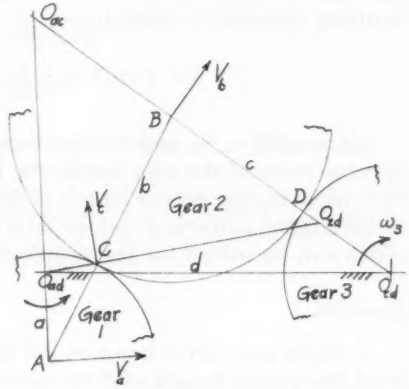
Although angle ψ was arbitrarily chosen in Fig. 1*a*, it is equally valid, in solving problems, to assume values for γ_1 and then get ψ, if needed, from the foregoing equations.

Velocities at various points of the linkage can also be obtained graphically as shown in Fig. 3*a*. Velocity V_a of point A is merely $a\omega_1$. Velocity of point C, as a point on gears 1 and 2, can be had by the usual kinematic construction. The directions of links a and c are extended to intersection at O_{ac} which is the instant center for link b. Then velocity V_b for point B as a point on link b and gear 2 is found by the usual construction. The velocities for two points on gear 2 are now known. The instant center O_{2d} for gear 2 is located at the intersection of the perpendiculars drawn at B and C. In the figure, O_{2d} is located on the side of D towards B. Gear 3 is thus turning counterclockwise, or in the same direction as driving gear 1. The lengths of the links are such as to cause this statement to be true for all other positions of the driving link.

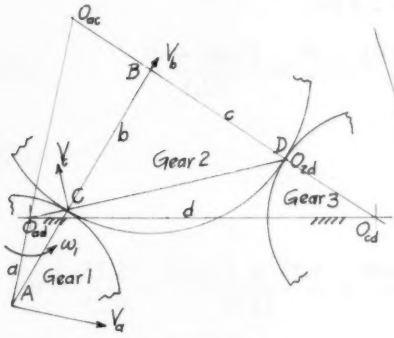
In Fig. 3*b*, link *d* has been lengthened so that center O_{2d} lies on the side of *D* towards O_{ed}. Gear 3 is thus turning oppositely or clockwise. Before and after this reversal of direction, the velocity of gear 3 must pass through zero. Positions of the links for this to occur are shown in sketches (*c*) and (*d*) where center O_{2d} coincides with *D*.



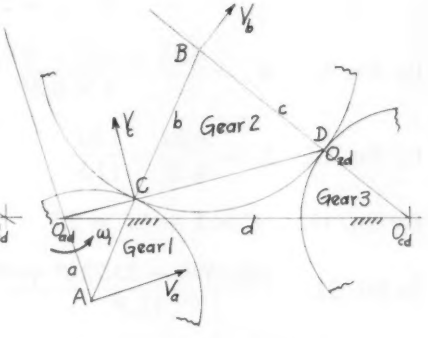
(*a*) Link *d* is sufficiently short so that gear 3 will not come to stop or reverse.



(*b*) Link *d* is sufficiently long so that gear 3 will reverse its motion.



(*c*) Linkage of sketch (*b*) in position for zero velocity of gear 3.



(*d*) Linkage of sketch (*b*) in position for zero velocity of gear 3.

FIG. 3. Reversing and non-reversing three-gear drives.

In some applications it is necessary for the output shaft to come to a full stop, but not reverse its motion, before proceeding further. The lengths of the links must be such as to cause ω_3 in Eq. 19 to be zero. In addition $\cos\theta$, for ω_3 equal to zero, must be single-valued and not double-valued as is the case in Fig. 3*c* and *d*. By the first of these

conditions, Eq. 19 gives the following quadratic in $\cos\theta$:

$$K_2^2(1 - C^2)\cos^2\theta + 2K_1K_2\cos\theta + K_1^2 + K_2^2C^2 - 1 = 0. \quad (21)$$

In order to be single-valued, the discriminant of this equation must be zero. The square of the coefficient of the cosine term is set equal to four times the coefficient of $\cos^2\theta$ times the absolute term. The resulting equation reduces to

$$K_1^2 + (C^2 - 1)K_2^2 - \frac{C^2 - 1}{C^2} = 0. \quad (22)$$

Since radii r_1, r_2, and r_3 must be maintained commensurate with the circular pitch of the gear teeth, the linkage for $\omega_3 = 0$ can be adjusted by computing the proper length of link d. At this point it is convenient to substitute numerical values into Eq. 22. A quadratic in d^2 results which can be solved for the proper value of d.

Example 2

A three-gear drive has $a = 0.8$ in., $r_1 = r_3 = 1$ in., and $r_2 = 1.5$ in. Find the proper length of d to cause the output shaft to come to a stop but not reverse. Find the value of θ at which this event occurs.

Solution

$$b = c = 2.5 \text{ in.}$$

By Eq. 9, $$K_1 = \frac{6.25 + 6.25 - 0.64 - d^2}{2 \times 6.25} = \frac{11.86 - d^2}{12.5} \quad (23)$$

By Eq. 10, $$K_2 = \frac{0.8d}{6.25} = \frac{1.6d}{12.5} \quad (24)$$

By Eq. 17, $$C = 1 + 1.5 = 2.5$$

In Eq. 22, $$\frac{140.6596 - 23.72\,d^2 + d^4}{12.5^2} + \frac{5.25 \times 2.56\,d^2}{12.5^2} - \frac{5.25}{6.25} = 0$$

or

$$d^4 - 10.28d^2 + 9.4096 = 0.$$

Then

$$d^2 = 9.26432$$

and

$$d = 3.04373 \text{ in.}$$

In Eq. 23. $$K_1 = 0.20766$$

In Eq. 24, $K_2 = 0.38960.$

Substitution into Eq. 21 gives the following quadratic in $\cos\theta$.

$$\cos^2\theta - 0.20305\cos\theta + 0.01031 = 0.$$

This has the solution

$$\cos\theta = 0.101523$$

or

$$\theta = 275.827°.$$

It is seen that a small adjustment in the length of d of Example 1 will give a mechanism in which gear 3 will exactly come to a stop.[4]

[4] Additional reading on the three-gear drive can be found in the following: J. A. BEGGS, "Mechanism," New York, McGraw-Hill Book Co., Inc., 1955, p. 50; A. RAPPAPORT, "Kinematics of Intermittent Mechanisms," *Product Engineering*, Vol. 21, p. 120 (1950).

MEDAL DAY PROCEEDINGS AT THE FRANKLIN INSTITUTE

October 21, 1959

RECEPTION AND DINNER

On Wednesday, October 21, 1959, approximately 420 Institute members and guests gathered to pay tribute to the men whose achievements had been recognized by the Committee on Science and the Arts during the past year. A reception honoring the 1959 Medalists was held at 6:30 P.M. in the lobby of Franklin Hall, and at 7:00 P.M. the guests assembled for dinner in the Hall. The Reverend Ernest A. Harding, D.D., Rector of Christ Church in Philadelphia, gave the invocation.

During dinner, Wynn Laurence LePage, President of the Institute, thanked the members of the Hostess Committee for their assistance at the reception and for the floral decorations. Mr. LePage proposed the following toast to Benjamin Franklin:

"We have come here this evening for the special purpose of honoring a small group of men whose achievements in the field of engineering and the physical sciences are deemed outstanding.

"We perform these functions in an atmosphere pervaded by the spirit of the one in whose honor this great Institute was founded one hundred and thirty-five years ago and in the inspiring memory of whom we carry on. Ladies and Gentlemen, rising and with glasses on high:

"To Benjamin Franklin!"

Dr. William G. Schmidt, Chairman of the Committee on Science and the Arts, introduced the following former Medalists who were present at the dinner.

FORMER MEDALISTS ATTENDING MEDAL DAY—1959

Year	Medalist	City
	CRESSON (Founded 1848)	
1903	G. H. Clamer	Philadelphia, Pa.
	CERTIFICATE OF MERIT (Founded 1882)	
1921	Joseph S. Hepburn	Philadelphia, Pa.
1942	Goodyear Tire and Rubber Company (Represented by A. G. Gracia)	Akron, Ohio
1942	Walter Larkin	Philadelphia, Pa.

Year	Medalist	City
	LONGSTRETH (Founded 1890)	
1896	The Pantasote Company (Represented by George T. Barks)	New York, N. Y.
1902	Williams, Brown and Earle (Represented by Peter Abrams)	Philadelphia, Pa.
1911	Joseph S. Hepburn	Philadelphia, Pa.
1921	W. Barton Eddison	Ardsley-on-Hudson, N. Y.
1922	Joseph F. Keller	New York, N. Y.
1923	Harry S. Parks	Pottstown, Pa.
1927	James F. Smathers	Poughkeepsie, N. Y.
1936	Frank N. Speller	Pittsburgh, Pa.
1937	John S. Haug	Philadelphia, Pa.
1947	Samuel Berman	Ozone Park, N. Y.
1948	Nicholas F. Arone	Philadelphia, Pa.
1955	Richard Y. Case	Philadelphia, Pa.
1958	George S. Crampton	Philadelphia, Pa.
	POTTS MEDAL (Founded 1906)	
1923	Albert W. Hull	Schenectady, N. Y.
1949	John W. Mauchly	Philadelphia, Pa.
1958	W. Nelson Goodwin, Jr.	New Hope, Pa.
	LEVY MEDAL (Founded 1923)	
1945	Rupen Eksergian	Philadelphia, Pa.
1948	William H. Cherry	Princeton, N. J.
1948	Jan Rajchman	Princeton, N. J.
	HENDERSON MEDAL (Founded 1924)	
1937	Rupen Eksergian	Philadelphia, Pa.
1954	C. Levon Eksergian	Philadelphia, Pa.
1955	Carleton K. Steins	Philadelphia, Pa.
	WETHERILL MEDAL (Founded 1925)	
1952	Albert J. Williams, Jr.	Philadelphia, Pa.
1954	Clarence N. Hickman	Jackson Heights, N. Y.
1955	R. A. Higonnet	Cambridge, Mass.
	CLARK MEDAL (Founded 1926)	
1958	Frederic O. Hess	Philadelphia, Pa.
	BROWN MEDAL (Founded 1938)	
1955	Charles S. Leopold	Philadelphia, Pa.
	CLAMER MEDAL (Founded 1943)	
1952	Cyril Stanley Smith	Chicago, Ill.
1958	Julian M. Avery	New York, N. Y.
	BALLANTINE MEDAL (Founded 1946)	
1952	Walter H. Brattain	Murray Hill, N. J.
1953	David G. C. Luck	Princeton, N. J.
1957	Robert M. Page	Washington, D. C.

MR. LEPAGE: "Thank you, Dr. Schmidt. And to you, Gentlemen. We welcome you warmly and sincerely and earnestly hope you will return again next year and for the many years ahead.

"As with most large organizations, educational, business or cultural, so with The Franklin Institute, a great deal of advisory work is carried on by committees. Thus, within our structure, we have Committees of the Board, various advisory committees and—one of the most active

and important—the Committee on Science and the Arts. It is of this committee I would speak for a moment. The Committee on Science and the Arts is one of the oldest committees of The Franklin Institute. Formerly known as the Committee on Inventions, back in the founding years, it was the committee most responsible for carrying out the Charter of the Institute. For, it was to this Committee the Board would turn for advice and counsel on knotty problems. A function of this Committee was to view, study, deliberate and advise inventors on the merits of their inventions. It went beyond that, too, for the Committee members were responsible for the encouragement of youthful inventors; encouragement to market or improve their inventions so that not only they, but all the public would benefit by their inventiveness.

"In 1834, the name of the Committee was changed to that by which we know it today, the Committee on Science and the Arts. Over the years the purposes of the Committee have substantially remained unchanged, but the methods by which inventions and discoveries are recognized have somewhat altered.

"For some years now, since about 1875, The Franklin Institute, following the recommendations of the Committee on Science and the Arts, has been awarding medals for achievement in the various fields of the physical sciences. The Committee on Science and the Arts, and its various sub-committees, some named after the medal awards they investigate—search out and examine the many candidates recommended for award. After lengthy study, a few scientists and engineers among the many who have contributed much to this country and the world are selected for recognition. We are gathered here tonight to recognize those few. But the hours, days and months of work that have gone before are to the credit of the gentlemen of the Committee on Science and the Arts. Gentlemen—we salute you and thank you. Your continuing devotion and service to The Franklin Institute does not go without recognition and appreciation."

STATED MEETING

MR. LEPAGE: "Today, Wednesday, October 21, 1959, marks the first Stated Meeting of The Franklin Institute of the State of Pennsylvania for this new season—and the first since May 20, 1959. The Minutes of that meeting were published in the June issue of the JOURNAL OF THE FRANKLIN INSTITUTE. If there are no additions or corrections to those minutes as published, I declare them approved as published.

"As has been our custom for many years, we take the steps necessary to welcome to membership in The Franklin Institute, the Franklin Medalist of the year. At the May, 1959, meeting of The Franklin Institute, Professor Hans Albrecht Bethe, the Franklin Medalist for 1959, was recommended and unanimously approved as Honorary Member of The Franklin Institute effective today. Dr. Bethe, will you come

forward? It is my pleasure, sir, to confer on you Honorary Membership in The Franklin Institute of the State of Pennsylvania, with all the privileges concomitant with that honor. Your membership certificate and cards, sir, and a warm welcome into this great organization.

"At this point in our program, it is my pleasant duty to call upon the Executive Vice President of The Franklin Institute, who will tell of some of our plans for the year ahead. Dr. John S. Burlew."

REPORT OF THE EXECUTIVE VICE PRESIDENT

DR. BURLEW: "Mr. LePage, distinguished Medalists, Ladies and Gentlemen: The contacts which different persons have with The Franklin Institute give them diverse impressions about its activities. Those who know only the public expressions of its program may not be aware of all that takes place here in this living memorial to Benjamin Franklin. Therefore, in telling you of the Coming Events at the Institute, I shall attempt to give you a general view of its activities.

"First, of course, are the evening meetings held for members of the Institute and their guests, which many of you have attended in the past. Let me mention a few of the more than twenty such meetings scheduled for this season. Many of them are Joint Meetings with local technical societies.

"The meeting next Wednesday will be unusual in that it has been arranged as a tribute to 'Ports of Philadelphia Day.' General E. C. Itschner, Chief of Engineers, United States Army, will speak about 'Franklin's River Today—The Story of the Deep Delaware.'

"On November 18, the Vermilye Medal, awarded biennially to an individual 'in recognition of outstanding contribution to the field of industrial management,' will be presented to George Romney, President of the American Motors Corporation. This same meeting will commemorate the One-hundred Thirty-fifth Anniversary of the founding of The Franklin Institute.

"At the Stated Meeting on December 16, Dr. W. F. G. Swann, who retired recently after thirty-two years of service as Director of the Institute's Bartol Research Foundation, will be honored. His address, entitled 'The Science of Yesterday, Today and Tomorrow,' is expected to match the many others Dr. Swann has given here, all of which have been so well received by his wide circle of friends.

"For the Annual Meeting in January, the Institute is very fortunate to have as the speaker Dr. Chauncey D. Leake, President of the American Association for the Advancement of Science, whose address will be entitled 'Our Unbiased Biad'—referring to our current effort to bring into balance the sciences and the humanities.

"These evening lectures are a part of the Institute's educational program. That program also includes visits by organized classes from

the public, parochial and private schools in Philadelphia and suburban communities. We have scheduled an even more extensive group of lecture-demonstrations in both the Planetarium and the Lecture Hall than last year, when over 150,000 school children came here to supplement the regular science instruction given in the schools.

"Another aspect of the Institute's educational efforts is the provision of a stimulus for the gifted youngster with special interest in science. The Science Museum, itself, is a powerful stimulus to such young people. In addition, the Institute has conducted a Workshop Program for the past eight years. This year 177 boys and girls have registered for the series of six classes which began last week.

"In the Science Museum each exhibit is in a sense an 'event.' The newest one, which I am sure will interest all of you, is a demonstration of the principles of aerodynamics. A Collins Smoke Tunnel, installed this fall in the Hall of Aviation through the generosity of a number of friends of the Institute, makes vividly clear to the visitor the interaction between an air stream and the parts of an airplane.

"These educational activities of The Franklin Institute are familiar to a wide group of persons in the Greater Philadelphia area. Less well known, but equally important in the over-all program of the Institute, are its research activities. These take place in the Laboratories for Research and Development and in the Computing Center here in Philadelphia, and at the Bartol Research Foundation located at Swarthmore.

"Events in these three departments of the Institute are important to the staff and to the sponsors of the projects performed there, even though they are not generally known to the public. Occasionally public notice is given one of them, as will occur next week when the First International Symposium on Gas-Lubricated Bearings takes place in Washington, D. C. This Symposium has been organized by the Friction and Lubrication Branch of The Franklin Institute Laboratories for the Office of Naval Research in cooperation with a number of other Government agencies. It is part of a project in the Institute concerned with the development of the technology of gas-lubricated bearings.

"A similar event will be a Symposium to be held at the Institute next February in cooperation with the Philadelphia Section of the American Nuclear Society on the subject of 'Gas-Cooled Nuclear Reactors.' The discussions at that Symposium will be of special concern to the three Branches of the Laboratories that are devoted to different aspects of nuclear engineering.

"Throughout the year many more events will occur at The Franklin Institute—each reflecting the imaginative planning and conscientious follow-through of members of the team. Especially important are the contributions of the heads of the departments of the Institute. Therefore, in closing I would like to mention the recent appointment of three

new department heads. Mr. Robert W. Neathery became Director of the Science Museum on August 1; Dr. Martin A. Pomerantz became Director of the Bartol Research Foundation on September 1; and Mr. John P. McGowan has accepted an appointment to the new position of Director of the Library, effective December 1. I feel we can look to these gentlemen to continue the tradition of their predecessors in providing a continuous sequence of important events in educational activities and research and development in the years ahead."

MR. LEPAGE: "Thank you, Dr. Burlew.

"Last year I was standing here for the first time as President of The Franklin Institute, and I confessed to you then that I was more than a little nervous, and perplexed as to what I should say. A year later, perhaps not as nervous, I am still perplexed as to what I can contribute, in the way of words, to this assemblage of scientists, engineers, business men—some my seniors in years, some my juniors, but these only in years. Since this time last year, much has happened in the world. We have seen more satellites launched; we have heard of greater progress in the curing and treatment of the ills of the bodies and minds of men; we have added two more stars to our flag; we have seen devices hurled through God's Firmament to another of His celestial bodies and we have seen the development and perfection of missiles that can fly from our land to the lands of another people. And, that those missiles may not be launched with warheads from here or from there, we are currently witnessing the meetings of leaders of nations—endeavoring to find common grounds of understanding.

"But here tonight, we are gathered in peaceful assembly to recognize, without regard to nation or political ideology, men of goodwill, whose efforts have been for the good of all mankind. We are here to meet and honor men who have helped push back the curtains of ignorance and darkness, who have helped us take one more step toward a fuller knowledge of the Universe we live in. And in being here we prove again that inherently in man the forces of peaceful cooperation and understanding are more potent than those of war and that there are many bases of mutuality and international cooperation among men of science and letters, which can and do have a profound influence upon the sustained peace of the World.

"I have attended many of these Medal Award ceremonies and now I am about to take part, for the second time, in presenting the tokens of recognition to scholars of this unusual Twentieth Century. And in presenting to these deserving men the awards of this venerable institution, I feel that not only are we honoring them, but they are honoring us—in the privilege of knowing them and meeting with them here for a little while.

"As I have indicated, the awards of The Franklin Institute are

given regardless of nationality, race or creed, for unusual and meritorious work in the physical sciences.

"The recipient of each award for 1959 is sponsored by the Chairman of that committee within the Committee on Science and the Arts, which recommended him for the award. Each sponsor will present the recipient and read his citation."

PRESENTATION OF AWARDS

PRESENTATION OF THE ELLIOTT CRESSON MEDALS

(Founded in 1848)

This Medal is awarded for discovery or original research, adding to the sum of human knowledge, irrespective of commercial value; leading and practical utilizations of discovery; and invention, methods or products embodying substantial elements of leadership in their respective classes, or unusual skill or perfection in workmanship.

To John Hays Hammond, Jr., President and Treasurer, Hammond Research Corporation, Gloucester, Massachusetts; to Henry C. Harrison, Consulting Engineer, Port Washington, New York; and to Irving Wolff, Vice-President, Research, Radio Corporation of America Laboratories, Princeton, New Jersey.

MR. LEPAGE: "The Chair recognizes Mr. Sidney T. Mackenzie."

MR. MACKENZIE: "Mr. President, I present John Hays Hammond, Jr., for an award.

"Dr. Hammond has devoted his life, since graduation from the Sheffield Scientific School in 1910, to the field of electronics, principally, radio. Out of the more than 600 patents that can be credited to him and his staff, some 44 can be considered major ones. Of this impressive list, undoubtedly, the most important contribution made by Dr. Hammond is that of remote control by radio of high speed moving objects.

"The application of radio control to a high-speed vessel that would resist deviation by wind and waves and at the same time would be free from jamming or taking over by an enemy was first made by Dr. Hammond on his yacht 'Natalia' in 1914. This control was in effect a gyro-stabilized automatic pilot under radio control. Its principle has been perfected and applied to radio controlled vessels, such as, target ships, target aircraft and controlled missiles.

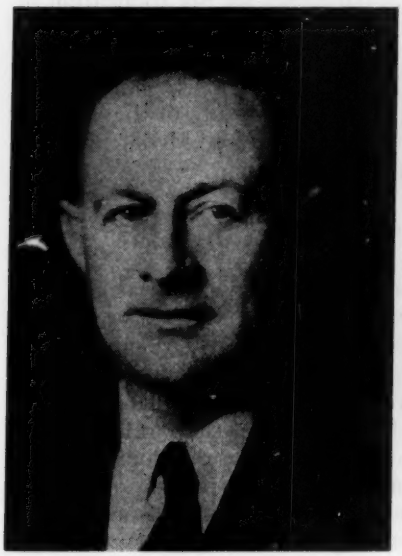
"Dr. Hammond holds basic patents on target seeking mechanisms. During 1912, he demonstrated his 'electric dog' which would 'home' on an electric light source.

"Applying his versatile imagination to other fields, we find that the Thermite incendiary bomb was his invention and that he contributed much to the improvement of the piano and the pipe organ.

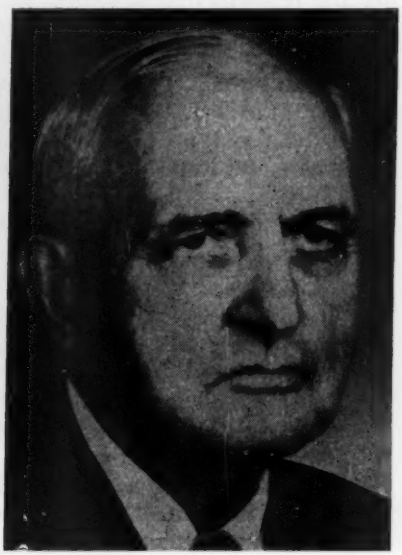
"I present John Hays Hammond, Jr., of Gloucester, Massachusetts, as a candidate for an Elliott Cresson Medal, 'In consideration of his many inventions and developments in the fields of radio and electronics; and, particularly, for his pioneering contributions and leadership in the invention, design and development of remote radio control for vehicles moving on or under the water.'"

MR LEPAGE: "Thank you, Mr. Mackenzie."

"Dr. Hammond, as President of The Franklin Institute of the State of Pennsylvania, I present to you this Elliott Cresson Medal and the Certificate and Report which accompany it."



John Hays Hammond, Jr.



Henry Charles Harrison

"The Chair recognizes Mr. Knox McIlwain."

MR. MCILWAIN: "Mr. President, I present Henry Charles Harrison for an award.

"Outside of the Bell Telephone System, Mr. Harrison is best known for his 'Matched Impedance' concept. This idea revolutionized sound reproduction and affected many other relationships.

"The concept of wave energy flow in mechanical or electromechanical systems through properly related elements was first developed by Mr. Harrison in his work on the carbon button transmitter. To overcome

some of its defects, he suggested obtaining best performance by matching the impedance starting with the feeble air vibrations, through the complex mechanical system, and finally into the electrical circuit. He later successfully applied the matched impedance philosophy to such devices as telephone receivers, ringers, microphones, horns, loudspeakers, recorders, and vibration-free mechanical driving systems.

"Other less widely known achievements include a mercury switch, wire spring relays, cross-bar switches, lever operated keys, stepping switches, multicontact relays, and a tuned-reed selector.

"I present Henry C. Harrison, of Port Washington, New York, as a candidate for an Elliott Cresson Medal, 'In consideration of a lifetime of prolific invention, which has resulted in new developments or in fundamental improvements in various fields of telephony and in recording and reproduction of sound, and in particular for his application of the matched impedance principle to mechanical and electromechanical devices.' "

MR. LEPAGE: "Thank you, Mr. McIlwain.

"Mr. Harrison, on behalf of The Franklin Institute of the State of Pennsylvania, I present to you an Elliott Cresson Medal and the Certificate and Report which accompany it.

"The Chair recognizes Mr. J. Philip Evans."

MR. EVANS: "Mr. President, I present Irving Wolff for an award.

"In his long and distinguished career with the Radio Corporation of America, Dr. Irving Wolff has made major contributions in many fields of electronics, radio communication and radio reconnaissance.

"His personal contributions as well as an inspired leadership of his colleagues led to developments which were of prime importance to the science and art of electronics; as for example, the development of a radar drift indicator which contributed greatly to safety of aircraft and navigation control, and the development of a radio altimeter of the frequency modulated type which made possible the extension of the range of the device and represented a basically important advance in this field.

"His contribution in the field of radar for the determination of aircraft altitude centered around the development of a system which greatly increased the accuracy of detection; while in the field of radar location, Dr. Wolff developed a number of basic ideas which were patented and which have resulted in the development of a three dimension location indicator.

"He joined the Research Staff of the Radio Corporation of America in 1924 and during World War II supervised the activities of a number

of research groups conducting radar research for the military agencies. In 1946 Dr. Wolff was appointed Director of the Radar Tube Research Laboratory of RCA, Director of Research in 1951 and in 1954 was elevated to the position of Vice President for Research. He received the Institute of Radio Engineers Fellow Award in 1942 for 'Basic research in centimeter-wave radio and application of it to the development of navigation instruments.' He is also a recipient of the Distinguished Public Service Award of the United States Navy for pioneer research in Radar.

Irving Wolff

"I present Irving Wolff, of Princeton, New Jersey, as a candidate for an Elliott Cresson Medal, 'In consideration of his many important contributions to the science of electronics as evidenced by his productiveness over a period of many years in many diverse fields such as acoustics, optics, radio, infrared detection and radio frequency heating and especially in view of his pioneering work in the centimeter wave field and his contributions to microwave radar development.' "

MR. LEPAGE: "Thank you, Mr. Evans.

"Dr. Wolff, on behalf of The Franklin Institute of the State of Pennsylvania, I present to you an Elliott Cresson Medal and the Certificate and Report which accompany it."

PRESENTATION OF THE EDWARD LONGSTRETH MEDALS

(Founded in 1890)

This Medal is awarded for inventions of high order and for particularly meritorious improvements and developments in machines and mechanical processes.

To David M. Potter, Jr., President, Potter Aeronautical Corporation, Union, New Jersey; and to Jacob Rabinow, President, Rabinow Engineering Company, Inc., Washington, D. C.

MR. LEPAGE: "The Chair recognizes Mr. Lionel F. Levy."

MR. LEVY: "Mr. President, I present David M. Potter, Jr., for an award.

"The turbine-type flow meter, that is a turbine rotor mounted in a pipe through which liquid is flowing, is rather old in the art and has been used quite successfully on many applications. It has been subject to serious limitations in sensitivity and accuracy by the drag load of the earlier form of mechanical response systems which were geared to the turbine rotor and also by the bearing friction and wear of the turbine bearing parts which are subject to immersion in the fluid which is being metered. The development of the magnetic electric tachometer or revolution counter coupled with electric amplification and calibration equipment has solved the former problem, and a unique solution for the second difficulty was developed by Mr. Potter.

"This he did by balancing the pressure forces on each end of the turbine rotor so that it is free floating. This was done by using the Venturi Tube principle. A cone, tapering away from the rotor, is fixed before and behind the floating rotor and a small space allowed between, so that unbalanced forces will move the rotor into balanced position.

"I present David M. Potter, Jr., of Union, New Jersey, as a candidate for an Edward Longstreth Medal, 'In consideration of the increased sensitivity and greater accuracy of an improved type of rotary flow meter developed by a unique application of the Venturi principle to balance the end thrust force to produce a free floating rotation free from the drag of end thrust.' "

MR. LEPAGE: "Thank you, Mr. Levy.

"Mr. Potter, on behalf of The Franklin Institute of the State of Pennsylvania, I present to you an Edward Longstreth Medal and the Certificate and Report which accompany it."

MR. LEPAGE: "The Chair recognizes Mr. Clarence B. Campbell."

MR. CAMPBELL: "Mr. President, I present Jacob Rabinow for an award.

"In 1947, Mr. Rabinow invented a new magnetic-type clutch to transmit and control torque. His clutch responds to electric current passed in series through parallel clutch discs whose interspace contains fine magnetic particles. These particles compact and adhere to the clutch faces when energized, developing torque forces which vary in magnitude with the electric energy input. The inventor showed that slip operation was smooth, deterioration negligible and response extremely rapid.

David M. Potter, Jr.

Jacob Rabinow

"His magnetic particle clutch principle is now used widely. For example, such clutches serve on naval ships to combine power transmission with precise output-shaft speed control; they are built into servomechanisms for some digital computers, automatic pilots and machine tools; they are used abroad for automotive clutches and automatic transmissions. Current development is directed toward similar automotive applications in our domestic small car field.

"I present Jacob Rabinow, of Takoma Park, Maryland, as a candidate for an Edward Longstreth Medal, 'In consideration of his invention of the Magnetic Fluid Clutch, a new concept of controlled coupling, of particular importance where smooth slip operation and/or extremely rapid response is essential.' "

MR. LEPAGE: "Thank you, Mr. Campbell.

"Mr. Rabinow, on behalf of The Franklin Institute of the State of Pennsylvania, I present to you this Edward Longstreth Medal and the Certificate and Report which accompany it."

PRESENTATION OF THE HOWARD N. POTTS MEDAL

(Founded in 1906)

This Medal is awarded for distinguished work in science or the arts; important development of previous basic discoveries, inventions or products of superior excellence or utilizing important principles.

To George W. Morey, Chemist, United States Geological Survey, Washington, D. C.

MR. LEPAGE: "The Chair recognizes Dr. Joseph S. Hepburn."

DR. HEPBURN: "Mr. President, I present George W. Morey for an award.

George W. Morey

"The researches of Morey were planned in 1927 and begun in 1932. At that time no glasses containing oxides of the rare earths and related elements of high atomic number were known with the exception of a didymium glass. As a result of his researches numerous glasses were produced derived from the rare earths, containing little or no silica, and characterized by a high index of refraction and a low dispersive power. During the period of World War II, over 125,000 pounds of these glasses were manufactured. They have been of especial value in the design and construction of optical apparatus for military purposes, and in lenses for aerial and other cameras. These glasses now are manufactured in the United States, Germany, England, and Japan.

"I present George W. Morey, of Washington, D. C., as a candidate for a Howard N. Potts Medal, 'In consideration of his researches culminating in the production on a commercial scale of glasses composed of rare earth oxides and little or no silica possessing a high index of refraction and a low dispersive power.' "

MR. LEPAGE: "Thank you, Dr. Hepburn.

"Dr. Morey, on behalf of The Franklin Institute of the State of Pennsylvania, I present to you this Howard N. Potts Medal and the Certificate and Report which accompany it."

PRESENTATION OF THE GEORGE R. HENDERSON MEDAL

(Founded in 1924)

This Medal is awarded for meritorious inventions or discoveries in the field of Railway Engineering.

To the Electro-Motive Division of the General Motors Corporation, LaGrange, Illinois.

MR. LEPAGE: "The Chair recognizes Mr. H. L. Decker."

MR. DECKER: "Mr. President, I present Cyrus R. Osborn, Executive Vice-President, General Motors Corporation, for an award to the Electro-Motive Division of that Corporation.

Cyrus R. Osborn

"General Motors Corporation, through its vast program of research and development, covering such phases as pioneering in the American development of Diesel engines and their components, as well as electric transmissions and control systems, introduced to American railroads a basic locomotive concept and initiated a policy of constant improvement in design, manufacturing, production, servicing facilities for railroads, merchandising and education of maintenance personnel. All of this resulted in the mass production of dependable and efficient Diesel-electric passenger, freight and switcher locomotives. The contributions of General Motors Corporation in this field have culminated in the virtual abandonment of steam locomotive power in North America.

"I present Cyrus R. Osborn, of General Motors Corporation, on behalf of their Electro-Motive Division, of LaGrange, Illinois, as a candidate for the George R. Henderson Medal, 'For the conception and demonstration of the feasibility of utilizing Diesel-electric locomotives on a wide scale in this country and for the design, development, standardization, production and servicing of these locomotives.' "

MR. LEPAGE: "Thank you, Mr. Decker.

"Mr. Osborn, on behalf of The Franklin Institute of the State of Pennsylvania, I present to you, representing the Electro-Motive Division of the General Motors Corporation, this George R. Henderson Medal and the Certificate and Report which accompany it."

PRESENTATION OF THE JOHN PRICE WETHERILL MEDALS

(Founded in 1925)

This Medal is awarded for discovery or invention in the physical sciences or for new and important combinations of principles or methods already known.

To Robert B. Aitchison, Consultant, Linde Company, Division of Union Carbide Corporation, New York, New York; to Clarence Zener, Director, Research Laboratories, Westinghouse Electric Corporation, Pittsburgh, Pennsylvania; and to A. T. James, Research Biochemist, National Institute for Medical Research, London, England; A. J. P. Martin, Director, Abbotsbury Laboratories, Limited, Elstree, Hertfordshire, England; and R. L. M. Synge, Research Biochemist, Rowett Research Institute, Bucksburn, Aberdeenshire, Scotland.

MR. LEPAGE: "The Chair recognizes Mr. Walter J. Kinderman."

MR. KINDERMAN: "Mr. President, I present Robert B. Aitchison for an award.

"Our nation's position of economic world leadership is closely related to iron and steel production and this is often referred to as the very backbone of our industrial strength. Through the years the bulk of our

domestic iron ore has come from the fabulous Lake Superior region deposits, particularly from the rich Mesabi Range which is now nearing depletion. Surrounding and underlying the rich primary ore, however, are far greater supplies of extremely hard Taconite ores which heretofore could not be mined economically.

"The development of the so-called Jet-Piercing process of drilling blast holes in this hard Taconite has made this vast new source of iron economically available in tonnages to supply the future needs of the industry for a century. In addition this new mining method is equally applicable to the mining, quarrying and excavating of extremely hard rock deposits. It was recently used very successfully in some of the excavation work of the great St. Lawrence Waterway.

Robert B. Aitchison

Clarence Melvin Zener

"Although Jet Piercing represents the collective teamwork of a research organization as well as extensive field testing, the basic development ideas and the leadership culminating in the success of the process came largely from one man.

"I present Robert B. Aitchison, of Panama City, Florida, as a candidate for a John Price Wetherill Medal, 'In consideration of his initiation, development and practical application of the jet-piercing process as applied to open pit mining and quarrying and the economic importance of this development to the steel industry and to the nation.'"

MR. LEPAGE: "Thank you, Mr. Kinderman.

"Mr. Aitchison, on behalf of The Franklin Institute of the State of Pennsylvania, I present to you this John Price Wetherill Medal and the Certificate and Report which accompany it.

"The Chair recognizes Dr. Frederic Palmer, Jr."

DR. PALMER: "Mr. President, I present Clarence Melvin Zener for an award.

"Dr. Zener has published some ninety scientific papers dealing for the most part with both theoretical and experimental problems in the fields of solid state physics and metallurgy. In addition he has published a book entitled, 'Elasticity and Anelasticity of Metals' which is regarded as a definitive work in the field of solid state physics.

"The importance of internal friction in metals was appreciated first by Dr. Zener who made this a research project of a rich and productive kind. He showed how internal friction affects and is affected by cold-working, annealing and high temperatures.

"Dr. Zener's studies of elasticity in metals contributed much to our understanding of the effects of temperature and rate of strain on the mechanical properties of metals; the mechanism of fracture, especially of steel; the mechanism of work-hardening and the growth of spherical precipitates during the precipitation hardening of metals. His paper on 'The Micro-mechanism of Fracture' is a classic in this field.

"One of Dr. Zener's outstanding attributes is his phenomenal ability to stimulate the work of others. He has brought to complex problems in physics, chemistry, and metallurgy simple, fundamental concepts that get to their core and make further progress possible.

"I present Clarence Melvin Zener, of Pittsburgh, Pennsylvania, as a candidate for a John Price Wetherill Medal, 'In consideration of his important contributions to the field of solid state physics and in particular to the physics of metals; and also of his unusual ability to stimulate important work on the part of others.' "

MR. LEPAGE: "Thank you, Dr. Palmer.

"Dr. Zener, on behalf of The Franklin Institute of the State of Pennsylvania, I present to you this John Price Wetherill Medal and the Certificate and Report which accompany it.

"The Chair recognizes Mr. Charles M. Cooper."

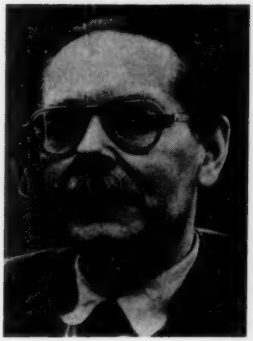
MR. COOPER: "Mr. President, I present, *in absentia*, A. J. P. Martin, R. L. M. Synge, and A. T. James for awards.

"From their coordinated investigations in the related fields of liquid-liquid phase chromatography and gas-liquid partition chromatography there has evolved the technique of gas chromatography—an analytical tool which has received a wide acceptance as a most effective means of separating diverse kinds of chemical constituents.

"I present, *in absentia*, A. J. P. Martin, R. L. M. Synge, and A. T.

James, all of Great Britain, as candidates each for a John Price Wetherill Medal, 'For the development of the theories underlying the manifold applications of gas chromatography to the quick and the precise analysis of minute quantities of mixtures of chemical components.' "

MR. LEPAGE: "Thank you, Mr. Cooper.



A. J. P. Martin

R. L. M. Synge

A. T. James

"I call upon The Honorable Geoffrey Aldington, Her British Majesty's Consul General. Mr. Aldington, on behalf of The Franklin Institute of the State of Pennsylvania, I present to you these John Price Wetherill Medals with their Certificates and Reports for Doctors James, Martin and Synge and request that they be forwarded to them at their homes in England and Scotland."

PRESENTATION OF THE WALTON CLARK MEDAL

(Founded in 1926)

This Medal is awarded to the "author of the most notable advance in knowledge or improvement in apparatus, or in method concerning the science or the art of gas manufacture or distribution or utilization in the production of illumination, or of heat, or of power."

To George G. Oberfell, Consultant, Phillips Petroleum Company, Bartlesville, Oklahoma.

MR. LEPAGE: "The Chair recognizes Mr. Joseph Gray Jackson."

MR. JACKSON: "Mr. President, I present George G. Oberfell for an award.

"Natural gasoline is the name given to light hydrocarbons recovered from natural gas by compression or absorption. The material originally contains large quantities of dissolved gases and is rich in very low

boiling hydrocarbons such as propane and butane, whose presence gives rise to certain operating difficulties known as 'vapor-lock.' In the early days of the natural gasoline industry, the propane and butane were allowed to evaporate and were lost. These materials were found to be quite suitable for domestic fuel as well as ingredients for motor fuel.

"Dr. Oberfell's initial efforts were devoted to the design and development of equipment to effect bulk delivery of these gases known as liquefied petroleum, or LP, gases thus greatly reducing the cost of gas as delivered to the consumer. High-pressure tank cars were designed

George G. Oberfell

for railroad transportation and similar pressure tanks were developed for use on motor trucks as well as the familiar cylinder for home use. These efforts resulted in a safe, reliable, highly improved source of fuel for domestic use.

"I present George G. Oberfell, of Bartlesville, Oklahoma, as a candidate for the Walton Clark Medal, 'In consideration of his significant contributions to the development of liquid petroleum (LP) gas as a source of domestic fuel.' "

MR. LEPAGE: "Thank you, Mr. Jackson.

"Dr. Oberfell, on behalf of The Franklin Institute of the State of Pennsylvania, I present to you this Walton Clark Medal and the Certificate and Report which accompany it."

PRESENTATION OF THE FRANK P. BROWN MEDAL

(Founded in 1938)

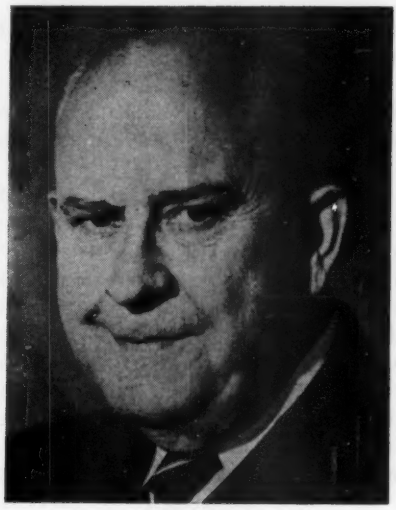
This Medal is awarded to inventors for discoveries and inventions involving meritorious improvements in the building and allied industries.

To Hardy Cross, Professor and Author, Virginia Beach, Virginia. (*Awarded Posthumously.*)

MR. LEPAGE: "The recipient of this award died on February 11, 1959, after this report had been adopted by the Committee on Science and the Arts, on January 14, 1959.

"The Chair recognizes Mr. Herbert H. Swinburne."

MR. SWINBURNE: "Mr. President, I present, posthumously, Hardy Cross for an award.



Hardy Cross

"Over the past half century, Professor Cross was widely known for his contributions to the science and art of structural engineering. His methods of structural design procedure have been adopted throughout the world, giving designers a much greater insight into the stresses that actually take place in monolithic frames.

"I present, posthumously, Hardy Cross, of Virginia Beach, Virginia, as a candidate for the Frank P. Brown Medal, 'For his outstanding

career as a teacher, especially of engineering students during the past fifty years at Hampden-Sydney College, Brown University, the University of Illinois and Yale University; for his many contributions to professional publications during this time; and, finally, for his original and later supplementary papers on the Moment-Distribution Method of analyzing Indeterminate Structures.' "

MR. LEPAGE: "Thank you, Mr. Swinburne.

"In the absence of Mrs. Inez M. Borthwick, long-time friend and nurse for the late Hardy Cross—because of sudden illness—I call upon Dr. John S. Burlew, Executive Vice President of The Franklin Institute. Dr. Burlew, will you see that this Frank P. Brown Medal and the Certificate and Report are forwarded to Mrs. Borthwick so that she may add this award to the many awards and honors Professor Cross received in his lifetime."

PRESENTATION OF THE FRANCIS J. CLAMER MEDAL

(Founded in 1943)

This Medal is awarded at least once in five years for meritorious achievement in the field of Metallurgy.

To Morris Cohen, Professor, Physical Metallurgy, Massachusetts Institute of Technology, Cambridge, Massachusetts.

MR. LEPAGE: "The Chair recognizes Mr. A. M. Bounds."

MR. BOUNDS: "Mr. President, I present Morris Cohen for an award.

"Commonly, medals of The Franklin Institute are awarded in recognition of a single notable achievement or invention in the field of the arts and sciences. Dr. Cohen's researches and writings have brilliantly covered so wide a field of physical metallurgy that it is extremely difficult to single out a particular piece of his research and cite it as being more deserving of commendation than his work in many other fields reported in his more than eighty published papers.

"Dr. Cohen has made an outstanding contribution to the understanding of the basic mechanisms involved in the martensitic reaction by which steel is hardened. Although this process has been carried out industrially for many years, it was his basic work in measuring the kinetics of the martensitic transformation and in clarifying the diffusionless nature of the reaction which led to an understanding of how the martensitic phase is nucleated from the parent structure. His investigations on martensite have provided a framework for the research required to improve the properties of many of the important materials used in modern advanced technology.

"From the scientific standpoint, however, these researches in the broad field of heat treatment of the highly alloyed steels cannot be put

above his researches in the field of pure physical metallurgy of both ferrous and non-ferrous metals.

"I present Morris Cohen, of Cambridge, Massachusetts, as a candidate for the Francis J. Clamer Medal, 'For his assiduous investigation and lucid exposition of the physical metallurgy of the heat treatment of steels as well as for a continuously high order of excellence of researches and publications covering the entire field of physical metallurgy.' "

MR. LEPAGE: "Thank you, Mr. Bounds.

"For The Franklin Institute of the State of Pennsylvania, I present to you, Dr. Cohen, this Francis J. Clamer Medal and the Certificate and Report which accompany it."

Morris Cohen

PRESENTATION OF THE STUART BALLANTINE MEDALS

(Founded in 1946)

This Medal is to be awarded in recognition of outstanding achievement in the fields of Communication and Reconnaissance which employ electromagnetic radiation.

To A. Hoyt Taylor, Formerly Chief Consultant, United States Naval Research Laboratory, Washington, D. C.; and to Charles H. Townes, Professor, Physics, Columbia Radiation Laboratory, Columbia University, New York, New York.

MR. LEPAGE: "The Chair recognizes Mr. Alan C. Byers."

MR. BYERS: "Mr. President, I present, *in absentia*, Albert Hoyt Taylor for an award.

"When Dr. Taylor began his scientific career the invention of the telephone was barely twenty-five years old, and Marconi's work had just begun. Dr. Taylor recognized the potentialities of worldwide communication by electromagnetic means and devoted his life to the development of these potentialities. When he retired after fifty very active years, radio communication had become so deeply embedded in our daily life that we are seldom conscious of its importance and rarely pause to consider how our progress would have been impossible without it.

"The major portion of Dr. Taylor's career was spent in military work, both as a Naval officer and as a Civil Servant, but his contributions were so fundamental that they influenced all phases of radio communications. Apparatus, operating techniques, frequency control, direction finding—all were improved by his work. Most important were his investigations of propagation phenomena which pointed the way to more reliable communications over long distances under varying ionospheric conditions.

"The progress of radar in this country was sparked by his early observations of radio reflection phenomena, and his vision of their military application. His support and direction of the work at the Naval Research Laboratory contributed to the timely attainment of useful radar systems.

"I present Albert Hoyt Taylor, of Claremont, California, as a candidate for the Stuart Ballantine Medal, 'For his more than fifty years of scientific work, which contributed notable advances in the technology of radio communications, particularly in the fields of radio wave propagation and of reliable communications systems.' "

MR. LEPAGE: "Thank you, Mr. Byers.

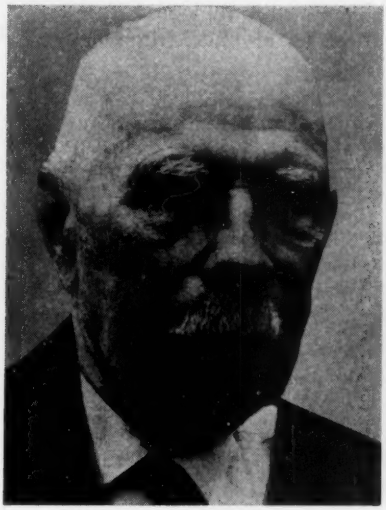
"I call upon Rear Admiral D. S. Fahrney, United States Navy, Retired, the Secretary to the Committee on Science and the Arts, and a former associate of Dr. Taylor. Admiral Fahrney, I present for forwarding to Dr. Taylor this Stuart Ballantine Medal and the Certificate and Report which accompany it.

"The Chair recognizes Mr. C. Raymond Kraus."

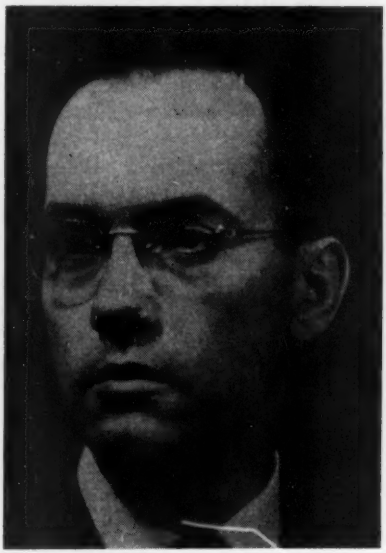
MR. KRAUS: "Mr. President, I present Charles H. Townes for an award.

"After World War II, Dr. Townes in his work in microwave spectroscopy conceived the idea of a new type of electromagnetic oscillator, based on the stimulated emission discovered by Einstein in 1917. Experimental evidence of the existence of sharply defined energy states in a beam of molecules was demonstrated by Stern and Gerlach in 1921 and extended by Rabi and his associates at Columbia University.

"Townes conceived a means of separating from a molecular beam one component beam containing molecules in an upper energy state and passing these molecules through a low loss cavity in which they could be made to interact with quanta emitted by preceding molecules without loss of the quantum which had induced the emission. Townes realized that quanta of radiation produced by stimulated emission are phase coherent with respect to stimulating quanta. Thus self-sustained oscillators could exist.



Albert Hoyt Taylor



Charles H. Townes

"The result was the first 'maser,' an acronym for 'Microwave Amplification by Stimulated Emission of Radiation.' The maser technique is a major breakthrough in the attainment of extremely precise frequency and timing references which have a frequency stability much greater than the astronomical bodies which are our present standards of time and frequency. Also, the maser technique has made possible the development of very low noise amplification devices which are opening up new vistas in extreme range communications and in radio astronomy.

"I present Charles H. Townes, of New York, New York, as a candidate for the Stuart Ballantine Medal, 'For his conception and demonstration of the feasibility of securing amplification and generation of high frequency radio waves by stimulated emission of radiation and for his invention of the maser which represents a new departure in electronic procedures and which has already led to many fruitful scientific and practical uses.' "

MR. LEPAGE: "Thank you, Mr. Kraus.

"Dr. Townes, for The Franklin Institute of the State of Pennsylvania, I present to you this Stuart Ballantine Medal and the Certificate and Report which accompany it."

PRESENTATION OF THE FRANKLIN MEDAL

(Founded in 1914)

This Medal is awarded annually to those workers in physical science or technology, without regard to country, whose efforts, in the opinion of the Institute, acting through its Committee on Science and the Arts, have done most to advance a knowledge of physical science or its applications.

To Hans Albrecht Bethe, Professor, Laboratory of Nuclear Studies, Cornell University, Ithaca, New York.

MR. LEPAGE: "The Franklin Medal is the highest award of The Franklin Institute. It was established in 1914 and was recently described by Federal Judge Palmieri as comparing 'on a world basis with the Nobel prize.' It is awarded to those workers in the physical sciences or technology, without regard to country, whose efforts, in the opinion of the Institute, have done the most to advance a knowledge of the physical sciences or their applications.

"The Chair recognizes Dr. Harvey A. Neville."

DR. NEVILLE: "Mr. President, I have the honor to present to you, as a candidate for the highest award of The Franklin Institute, the distinguished physicist Hans Albrecht Bethe.

"Dr. Bethe, a native of Germany, was educated in that country and began his career as a teacher of physics at the German universities. In 1935 he came to the United States to join the faculty of Cornell University where he is now John Wendell Anderson Professor of Physics and Director of the Laboratory of Nuclear Studies. He was Chief of the Theoretical Physics Division at the Los Alamos Laboratory, 1943–1946, has been a consultant to the United States Atomic Energy Commission since 1947, and is a member of the President's Science Advisory Committee.

"The scientific work of Dr. Bethe has received further recognition by the award to him of the A. Cressy Morrison Prize of the New York Academy of Science in 1938 and again in 1940, the United States Medal of Merit in 1946, the Draper Medal of the National Academy of Sciences in 1948, and the Max Planck Medal of the German Physical Society in 1955.

"Dr. Bethe is the author of several major treatises and texts in the fields of atomic and nuclear physics. He has published well over one

hundred research articles which by their range of topics indicate the remarkable versatility of his interests as well as his unusual energy.

"To illustrate the breadth and variety of his work in theoretical physics, the following are a few of the subjects treated in his scientific papers: the stopping power of matter for ionizing radiations, the diffraction of electrons by crystals, term separations in crystals, the physics of metals, long-range and short-range order, the photo-electric effect, theory of the Zeeman effect in the rare earth salts, the Lamb shift, relativistic processes of pair production, neutron resonances, the distinction between pi and mu mesons, nuclear binding forces, and energy production in stars.

Hans Albrecht Bethe

"It can be readily understood that the scientific contributions of Hans Bethe are primarily of interest to other physicists and are comprehensible only to them. It is not feasible, therefore, to present in popular language the significance of his work, or to explain to the general public its utmost importance in the progress of science.

"One of his accomplishments which is somewhat amenable to popular exposition is his brilliant elucidation of the thermonuclear processes responsible for energy production in stars. An acceptable theory for this phenomenon must account for both the enormous rate of energy production and for the very long period through which this output is sustained. Bethe showed how a series of nuclear transmutation reactions can meet these requirements with only hydrogen as the fuel, helium as the product, and carbon as the catalyst for the cycle of

reactions. This example merely illustrates how, through his own work and through his stimulating influence upon other scientists, Dr. Bethe has had a major role in the development of modern physical theory.

"Therefore, Mr. President, with this formal citation: 'For his many profound investigations which have contributed significantly to our understanding of the physical universe from nuclei of atoms to the interior of the stars,' I present to you, Hans Albrecht Bethe, of Ithaca, New York, as a candidate for The Franklin Medal."

MR. LEPAGE: "Thank you, Dr. Neville."

MR. LEPAGE: "Dr. Bethe, it gives me great pleasure to present to you The Franklin Medal and the Certificate and Report which accompany it. In receiving this Medal, you become one of a distinguished company of outstanding workers in the physical sciences, including Thomas Edison, Guglielmo Marconi, J. J. Thomson, Max Planck, Niels Bohr, Enrico Fermi and Donald Douglas, who are just a few of the recipients of The Franklin Medal. It is a privilege to welcome you to the ranks of the Franklin Medalists."

Mr. LePage then introduced Dr. Bethe as the speaker of the evening.

ADDRESS BY HANS ALBRECHT BETHE

(See page 501 of this JOURNAL for the text of Dr. Bethe's address.)

After Dr. Bethe had delivered his talk, Mr. LePage conveyed to him the thanks of the Institute, dismissed the assembly with a cordial word and declared the meeting adjourned.

FUNDAMENTAL PARTICLES *

BY

HANS ALBRECHT BETHE [1]

Mr. President, Ladies and Gentlemen: It is a great honor for me to be here as the evening's speaker as well as the recipient of the medal named after the patron of this Institute.

As long as men have thought about the world around them they have wanted to know what the world is made of. The Greeks, who started most of philosophy, thought about this and they came to the conclusion that there are two possibilities: either matter is divisible and more divisible into smaller and smaller parts, or else there must be some smallest part of matter which they called the atom.

You know that the Greek philosopher Democritus was the one who chose the right side. He chose the side of atoms. This was very good logic. It was not science. The science of the constitution of matter started just about in the time of Benjamin Franklin. The great chemists—Priestley, Lavoisier and others—discovered what matter was made of. They discovered that there are immutable parts of matter, immutable constituents which they called chemical elements, and they found that through all changes of appearance these chemical elements always remained the same.

If you take a piece of coal, which is a black and hard substance, and burn it with the help of oxygen, which is an invisible life-giving gas, you get another gas from it, nothing solid. You get something that is completely different from both the constituents—carbon dioxide. And, as you know, plants have the ability of reversing this process and of changing the carbon dioxide back into the elements oxygen and carbon. Through all of these transformations there remain the same amounts of carbon and of oxygen. This conservation of the elements was one of the greatest discoveries in the constitution of matter.

Chemists discovered subsequently a large number of elements and they discovered enough about the way these combine to become convinced that elements consist of atoms as Democritus had predicted on purely logical and not scientific grounds: all matter consists of atoms.

In the course of time we have learned to recognize 101 different species of atoms and we number them with an atomic number from 1 for hydrogen to 101 for mendelevium, named after the great Russian

* Delivered at the 1959 Medal Day Meeting of The Franklin Institute, October 21, 1959, in acceptance of the Franklin Medal.

[1] John Wendell Anderson Professor of Physics, Cornell University, Ithaca, N. Y.

scientist Mendeleev who brought some order into this great confusion and profusion of chemical elements.

A hundred and one different elements is a lot to swallow, and people were thinking very soon that there must be something simpler in matter, that matter couldn't be as complicated as that—101 completely different things; and about a hundred years ago Prout made the hypothesis that all elements are composed of simpler things. He was led to this supposition by the observation that the atomic weights of all elements are nearly whole numbers when expressed with the atomic weight of hydrogen as 1. He, therefore, assumed that hydrogen is the constituent of all the atoms. Nobody up to that time had discovered what we now call a fundamental particle. The first fundamental particle was discovered by J. J. Thomson about 1880 and I am proud to hear that J. J. Thomson was one of my colleagues in receiving the Franklin Medal. The particle he discovered was the electron.

The electron is a very small particle, as you know. It didn't fit into Prout's scheme of the world. Its weight is only about 1/2000 of the weight of the hydrogen atom. Nevertheless, we know that it is one of the fundamental constituents of matter which, in fact, is present in every atom.

The electron is so called because it has an electric charge, one unit of negative electric charge; and it is very fitting to say this here under the benign eyes of Benjamin Franklin who contributed so much to the knowledge of electricity. We know now that electric forces are the thing which holds matter together. Electric forces hold together the atom consisting of a positively charged nucleus and negatively charged electrons. It is the electric attraction between these particles which makes the atoms have the properties which they have; it is this electric attraction which is the basis of the laws of chemistry and of all the multiform variations of spectral lines and colors around us.

The electron was the first fundamental particle discovered. Since it had a negative charge, there had to be something positively charged in the atom and this was soon discovered, about 1910, by Lord Rutherford on the basis of researches on radioactivity. He found evidence that in the center of each atom there is a very massive, positively charged thing which he called the atomic nucleus. Each atom is characterized by the charge of its nucleus. If it has 59 units of charge, then 59 electrons will circle around that nucleus. The charge of the nucleus determines all the chemical properties of the atom because it determines the number of electrons in the atom.

At the same time Rutherford discovered that the atomic nucleus was the central part of the atom he also found, together with other researchers in the field, that the nucleus itself is not a fundamental particle because the radioactivity from which he had deduced his theory of the nucleus showed that nuclei transform into each other. The uranium

nucleus, a radioactive nucleus, emits other particles which are helium nuclei. The remains of the uranium nuclei transform into radium after a while, then into a gas called radon, then into other chemical elements, and finally into lead.

This radioactive transformation showed that the nucleus is not a fundamental particle, but consists of still smaller particles. What were they? It was natural to assume that the lightest nucleus, the nucleus of hydrogen, of atomic weight 1, was the constituent of which all other nuclei are built up. So, said Rutherford, let us say that all the mass of the nucleus is made up of hydrogen nuclei, which he called protons. There are just as many hydrogen nuclei in one given nucleus as the mass of the nucleus indicates, and then the charge is made right by the presence of enough electrons so that the total charge of the nucleus comes out correctly.

We had, then, the very nice and simple picture that there were two fundamental particles of nature: the proton, positively charged and heavy; the electron, negatively charged and very, very light.

True enough, this didn't explain very much of what was going on. One didn't know how these particles were held together in the nucleus and one knew only very vaguely, owing to the researches of Niels Bohr (another colleague of mine, as I proudly note), how the electrons go around the nucleus in an atom.

This state of ignorance was changed by quantum mechanics, a science which was based on the quantum theory of Max Planck and which was established in 1925 and '26. Quantum mechanics is a very wonderful thing. It tells you how electrons move around the nucleus in the atom. It tells you how the atom acquires its properties—spectroscopic, magnetic, chemical and other properties. It permits you to calculate with great precision, if you can solve the mathematics, all the energy levels of an atom, all the properties of an atom. It gave us a completely new concept of the motion of particles in small spaces, which is very different from the motion of big objects.

Quantum mechanics, while extremely constructive, also destroyed Rutherford's simple picture that there are only two elementary particles. It did this in a very remarkable way. The greatest refinement of quantum mechanics is relativistic quantum mechanics which incorporates the Relativity Theory of Einstein, that is, the theory of motion with high velocity. This relativity quantum mechanics makes it possible to predict very fine details of the spectra of atoms.

At the same time it predicted that there should not only be the negatively charged electron which Thomson had discovered, but that there should be another particle equal in mass, equal in all properties, equal in its motions, but having a positive charge instead of a negative one. This particle, known as the positron, was predicted by Dirac on the basis of his relativistic quantum mechanics in 1929.

In 1932 the great theoretical physicist Pauli, probably the most penetrating of all theoretical physicists in quantum mechanics, wrote a famous article in the *Handbuch der Physik* in which he said that Dirac's theory is beautiful, and that it explains all sorts of things, but it must be wrong because it predicts that there should be a particle like the electron having a positive charge. No such particle had been found and, therefore, the theory must be wrong.

This was written in 1932. In 1933 the particle was found. It was found by Anderson at the California Institute of Technology, in cosmic rays. It was a beautiful triumph of theoretical physics—the prediction of the existence of a new particle on purely theoretical grounds. It increased the family of fundamental particles, but it did far more because it was soon found, again in accord with theoretical predictions of Dirac's theory, that this positive electron, together with the familiar negative electron, can be produced out of pure energy.

The purest form of energy which we know is the light quantum which comes from the electric bulb or from the candles here and is received by our eyes. Light quanta are just forms of energy and everybody considered them just as energy until 1933. In 1933 it was discovered that this pure form of energy could change into a pure form of matter, namely, into a positive and a negative electron which until then rightly had been considered very definitely particles, very definitely constituents of matter.

Thus was proved Einstein's famous equivalence of energy and matter. Thus was proved something which I think physicists had not realized before, namely, that there is no real dividing line between a live quantum (electromagnetic energy) on one side and matter on the other side. Now some people who found this too revolutionary could take refuge in the idea that an electron is an awfully small particle, awfully light. Maybe, they thought the proton does not have this peculiar property of being able to be created out of nothing, out of energy.

Well, if people thought so they were again wrong. Theory again predicted—and theory, in fact, predicts it for any particle—that there must be a counterpart to the proton as well, a particle which behaves precisely in the same way as the proton, having precisely the same mass and the same magnitude of charge, but having a negative charge instead of a positive one.

It took an enormously long time, over twenty years, from the discovery of the positron to the discovery of the negative proton, the anti-proton. It was discovered at the University of California by Segrè and collaborators in 1955, using the big accelerator at that University.

Now we have already quite a profusion of particles and things are becoming quite complicated. More were to come; in fact, more had already come earlier. Quantum mechanics again said something else against poor Lord Rutherford. It said that he must be wrong in saying

that electrons are in the nucleus, because quantum mechanics said that you cannot confine any particle in a space less than h/mc where h is Planck's constant, m is the mass of the particle and c is the velocity of light.

Since the electron is a very light particle, its mass is very small and, therefore, it needs a large space; h/mc is the space in which you can confine a particle. It needs an enormous space, namely, 2×10^{-11} centimeters. That doesn't sound very large. However, it is a hundred times larger than the helium nucleus and, therefore, it could be concluded that electrons could not be inside any nucleus.

What, then, was in the nucleus? The answer was soon found—in fact, earlier than the positron—when Chadwick in 1932 discovered another particle, the neutron. The neutron is very much like the proton. It has just about the same mass. It has many other properties in common, but it has no charge. Now it was easy to think how to make up the nucleus because these particles, protons and neutrons, are heavy particles; therefore, you can confine them in a small space two thousand times less than the space in which you can confine the electrons and, therefore, it is easy to put them in the small nucleus.

The theory was advanced, then, that the nucleus is composed of protons and neutrons and we have every reason to believe that this theory is correct. It does not have all the quantitative triumphs that quantum mechanics has in the explanation of the outer part of the atom. Of course, it again uses quantum mechanics and it can explain in a general way, but not with exact numbers, the way atomic nuclei are made up.

We still have a problem. We know that electrons come out of nuclei in radioactivity; in fact, this is how Rutherford first established his theory of the nucleus. How can electrons come out of nuclei if there is no room in a nucleus for them? Are they inside the neutron? There is no chance of that because the neutron is even smaller than the nucleus. So we have to accept again a new concept of matter, namely, that particles can come out of another particle without having been in that other particle before. They are created at the moment at which they come out.

A proton can change into a neutron and a positive electron. A neutron can change into a proton and a negative electron. We see the electrons come out. The nucleons—neutrons or protons—remain in the nucleus and cannot be distinguished from a nucleon which was in the nucleus before.

Since 1932 and '33 a lot more has been learned about particles. There are twenty-three additional particles which I have not mentioned to you. There are neutrinos which are very peculiar particles, having no charge and no mass. There are mesons of various Greek initials—pi and mu— and the Latin initial K, and there are other particles which

we also consider as fundamental particles, called hyperons which are heavier than the nucleus. They have earned Nobel prizes for their theoretical predictors and for their experimental discoverers. There is not time to mention them, although they are all interesting.

We have a profusion, again, of thirty different particles and we again needed a Mendeleev to put some order into this profusion. His name was Gell-Mann, and he was able to sort these particles into various types, various groups, so that we have some idea how they fit together and how they are necessary for the structure of the universe. Most of these particles are very unstable and live only for times like 10^{-8} or 10^{-10} seconds.

All these particles exist and each of them can transform into two or three others, as the neutron can transform into a proton and an electron.

If there is nothing stable in this universe, what is fundamental? Is there anything that remains conserved when all these transformations take place? Yes, there is. First of all, there is electric charge. Electric charge always remains conserved. When you make particles, when you change one particle into others, the sum of the electric charges of the particles which you make must equal the electric charge of the initial one.

There is energy. Energy, as far as we can tell, always remains conserved. And there is at least one more conserved quantity which we call the nucleon number. It is true that one can change a proton into a neutron. We consider them modifications of the same particle which we call the nucleon. It is true that you can make a proton and an anti-proton, a negative proton, out of energy, but you can only make them in pairs; so we give to the nucleon, the proton or neutron, the nucleon number of $+1$ and to the anti-nucleon the nucleon number of -1. When we make things out of energy we start with nucleon number 0 and, therefore, we can only make a $+1$ together with a -1.

Similarly, a nucleon and an anti-nucleon can come together again and annihilate and give energy, but never can a proton and an electron come together and just give energy. This is extremely important and is different from what people believed back in the 1920's.

If it were possible that a proton and an electron could come together and make energy (which obviously is possible from the conservation of charge point of view because you have one positive and one negative charge), then matter as we know it could not exist. We could not exist and we could not be in this beautiful room because it could not exist. The Earth could not be there. We know that the Earth has lived for several billions of years and we know, furthermore, that there is not a great deal of energy being released inside the Earth. If protons and electrons could combine to make energy, then an enormous amount of energy would be released in the interior of the Earth all the time. There

is some energy released there, but that can be well accounted for by ordinary radioactivity.

We know, therefore, that certainly not more than 10^{-10} of the hydrogen atoms and other atoms in the Earth can have changed in this way during the life of the universe, which is about 10^{10} years. So if this process happens at all it must go incredibly slowly. We can never say in physics that something is absolutely absent or absolutely impossible on the basis of evidence. We can only say that if it occurs it must be extremely unlikely. We have every reason to believe that, in fact, this combination of electron and proton to make pure energy never happens.

So we have a few things which are conserved and one of them, and perhaps the most important, is electric charge. And, again, it is fitting to remember Benjamin Franklin who was one of the pioneers in the science of electricity which has benefited us not only in practical ways, but so much in the theoretical understanding of the world in which we live.

Thank you.

THE FRANKLIN INSTITUTE

MINUTES OF THE STATED MEETING

November 18, 1959

The Stated Meeting of The Franklin Institute was held at 8:15 P.M. in Franklin Hall, following a reception and dinner at which approximately 285 members and guests were present.

Mr. Wynn Laurence LePage, President, called the meeting to order and announced that the minutes of the October Stated Meeting had been printed in the November issue of the JOURNAL OF THE FRANKLIN INSTITUTE. There being no corrections or additions, the minutes were approved as published.

The President stated that on October 14, 1959 there were mailed to the Voting Members of The Franklin Institute certain proposed amendments to Article IV, Section 5 and Article VIII, Section 3 of the By-Laws. He called for a motion on these proposed amendments.* Mr. Richard T. Nalle presented the following resolution:

> RESOLVED, That Article IV, Section 5 and Article VIII, Section 3 of the By-Laws of The Franklin Institute of the State of Pennsylvania for the Promotion of the Mechanic Arts be amended to read as set forth as part of the notice to members dated October 14, 1959, and that these amendments become effective as of this date, November 18, 1959.

This motion was seconded by Mr. James H. Robins, and the President called for a vote. The motion was unanimously adopted.

The President announced that the meeting this evening celebrates the 135th Anniversary of The Franklin Institute and the 25th Anniversary of the edifice on the Benjamin Franklin Parkway and of this great Memorial Hall. He followed the progress of The Institute from 1824 to the present time, showing that the purposes of The Institute as conceived and established in 1824 were being carried out to the present day. President LePage said we will continue to live up to the charge of our history and in so doing we shall draw strength from the warm friends who surround us, the business, cultural and educational community, and the imillions of people who have come to depend upon The Franklin Institute as their source of ntellectual inspiration in matters scientific.

The President then called on Dr. Francis J. Chesterman, a member of The Institute's Board of Managers and Chairman of the Vermilye Medal Committee, who presented George Romney, President of the American Motors Corporation, as the Vermilye Medalist for 1959. Dr. Chesterman said the Medal is awarded biennially by The Franklin Institute "in recognition of outstanding contribution in the field of management." The President presented the Medal and the Citation to Mr. Romney and introduced him as the speaker.

Mr. Romney spoke inspiringly about the need for sound leadership in labor, industry and government, and through strong leadership, with proper balances, to provide preventive measures to meet national emergencies. In closing, Mr. Romney quoted Somerset Maugham, "A nation that wants anything more than freedom will lose its freedom, and the irony of it is, if it is comfort and security it wants, it will lose them, too."

Mr. Romney was given an enthusiastic and rising ovation. Mr. LePage spoke of how proud Benjamin Franklin would have been of Mr. Romney and how proud we were to have him with us on this occasion.

The meeting was adjourned at 9:45 P.M.

WILLIAM F. JACKSON, JR.
Secretary

* Amended By-Laws are on page 511.

NOTE: The entire proceedings will be published in the February 1960 issue of the JOURNAL.

MEMBERSHIP

The following elections were approved on November 5, 1959 by the Membership Committee by authority given to it by the Board of Managers.

ANNUAL

A. Ross Adams
James N. Ademino
Edward H. Altman
Abraham Asher
Selden D. Bacon
William L. Baker
Norman L. Barr
James M. Boyer
John H. Buck
Willard P. Davis
Robert I. Dinlocker
Noel E. Donlin
Paul K. Duffey
William Eismann, Jr.
Charles T. Engelman, Jr.
Robert D. Epperson
Aaron M. Fine
Daniel Geary
William T. Geiger
Richard J. Hall
F. DeWitt Kay, Jr.
George J. Kmecik
Frank F. Koblitz
Douglas N. Lapp
Alfred Lohmeier
David C. MacDowell
Samuel B. Magill
Henry Mancuso
Anthony James Mannino
Hector T. McVey
Quentin D. Mehrkam
Charles E. Mindnich
Charles D. Moberg
D. Cortlandt Montgomery
George R. Moreland
Earll R. Muddiman
Leonard Peller
Frank D. Peltier
William J. Peltz
Constantin Pervanis
Ralph C. Peters
Ovid S. Petrescu
Frank F. Pfeiffer
A. Henry Philips
Lloyd B. Phillips
Karl A. Pippart
David S. Plewes
Horace W. Potter
George W. Powell, Jr.
James A. Pratt
Joseph V. Radziul
Adolph K. Rapp
Joseph R. Raudenbush
Edward Robinson
Harvey Rosenbluth
Leon B. Rosseau
Abraham Schneider
George H. Schneller
Willard R. Seipt
Albert J. Sziarto
Thomas J. Taylor
Frank Tidona
Robert T. Vaughn
Theodore I. Veiock
Richard E. Waggener
Reginal L. Wakeman
Stuart G. Younkin

EDUCATOR

Alma D. Brink
Theodore H. Ingalls
H. B. Kostenbauder
Mrs. Fred M. Oberlander
Philip C. Prager
Murray M. Tuckerman

NON-RESIDENT

H. T. Friis

NECROLOGY

Mrs. John R. Avery '59
Russel L. Brinton '14
Robert Cameron Colwell '20
Arthur R. Earnshaw '35
Richard Grosholz '34
Joseph J. Hagan '47
Gen. George C. Marshall '47
Joseph C. Rosengarten, Jr. '36
Sir Henry Thomas Tizard '46

LIBRARY

The Committee on Library desires to add to the collections any technical works that members may wish to contribute. Contributions will be gratefully acknowledged and placed in the Library. Duplicates received will be transferred to other libraries as gifts of the donor. 427 items have been added the past month.

Photostat service. Photostat prints of any material in the collections can be supplied on request.

The Library and reading room are open from 9 A.M. until 5 P.M. on Mondays, Tuesdays, Thursdays, Fridays, and Saturdays; and 2 P.M. until 10 P.M. on Wednesdays.

RECENT ADDITIONS

AERONAUTICS

TRUITT, ROBERT WESLEY. Hypersonic Aerodynamics. 1959.

ASTRONOMY

CYR, DONALD LEE. Mars Revisited. 1959.
INTERNATIONAL ASTRONAUTICAL CONGRESS. VIII. BARCELONA, 1957. Proceedings. 1958.

CHEMISTRY AND CHEMICAL TECHNOLOGY

INTERNATIONAL UNION OF PURE AND APPLIED CHEMISTRY. Nomenclature of Inorganic Chemistry. 1958.
INTERNATIONAL UNION OF PURE AND APPLIED CHEMISTRY. Nomenclature of Organic Chemistry. 1958.
JORISSEN, WILLEM PAULINUS. Induced Oxidation. 1959.
ROCHLIN, ROBERT S. AND SCHULTZ, WARNER W. Radioisotopes for Industry. 1959.
SEYMOUR, RAYMOND BENEDICT. Hot Organic Coatings. 1959.
SOCIETY OF CHEMICAL INDUSTRY. Physical Properties of Polymers. 1959.
WATERS, WILLIAM ALEXANDER, ED. Vistas in Free-Radical Chemistry, Published in Memoriam to Dr. Morris S. Kharash. . . . 1959.

CIVIL ENGINEERING

HAMMOND, ROLT. Tunnel Engineering. 1959.

ELECTRICITY AND ELECTRICAL ENGINEERING

KNOLL, MAX. Materials and Processes of Electron Devices. 1959.
PENROSE, HAROLD ERNEST AND BOULDING, REGINALD SIDNEY HENRY. Principles and Practice of Radar. Ed. 6. 1959.
ZEINES, BEN. Servomechanism Fundamentals. 1959.

GENERAL

TIMOSHENKO, STEPHEN P. Engineering Education in Russia. 1959.

MANUFACTURE

BROWNELL, LLOYD EARL AND YOUNG, EDWIN HAROLD. Process Equipment Design; Vessel Design. 1959.
MINER, HARVEY D. AND MILLER, JOHN GUTHRIE. Exploring Patternmaking and Foundry. 1959.
SODERBERG, GEORGE A. Finishing Materials and Methods. Ed. 2. 1959.

MATHEMATICS

CASHWELL, E. D. AND EVERETT, CORNELIUS JOSEPH. A Practical Manual on the Monte Carlo Method for Random Walk Problems. 1959.
IRVING, J. AND MULLINEUX, N. Mathematics in Physics and Engineering. 1959.
WARREN, A. G. Mathematics Applied to Electrical Engineering. Ed. 2, rev. 1958.

MECHANICAL ENGINEERING

ARMSTRONG, L. V. AND HARTMAN, J. B. The Diesel Engine; its Theory, Basic Design and Economics. 1959.

METALLURGY

KEYSER, CARL A. Basic Engineering Metallurgy; Theories, Principles and Applications. Ed. 2. 1959.

METEOROLOGY

BATTAN, LOUIS J. Radar Meteorology. 1959.
PRIESTLEY, CHARLES HENRY BRIAN. Turbulent Transfer in the Lower Atmosphere. 1959.

MISSILES AND ROCKETS

PARKIN, CHARLES M., ED. The Rocket Handbook for Amateurs; an Illustrated Guide to the Safe Construction, Testing and Launching of Model Rockets. 1959.

PHYSICS

HAMILTON, JAMES. Theory of Elementary Particles. 1959.

HERZFELD, KARL FERDINAND AND LITOVITZ, THEODORE A. Absorption and Dispersion of Ultrasonic Waves. 1959.

MERCIER, ANDRÉ. Analytical and Canonical Formalism in Physics. 1959.

ROUSE, HUNTER AND INCE, SIMON. History of Hydraulics. 1957.

THOMSON, JOHN AND CALLICK, E. B. Electron Physics and Technology. 1959.

WANNIER, GREGORY H. Elements of Solid State Theory. 1959.

RADIATION

Radiation Hygiene Handbook; A Practical Reference Book Covering the Industrial, Medical and Research Uses of Radiation and Atomic Energy. . . . 1959.

RESEARCH

QUINN, JAMES BRIAN. Yardsticks for Industrial Research; the Evaluation of Research and Development Output. 1959.

SCIENCE

POPPER, KARL RAIMUND. The Logic of Scientific Discovery. 1959.

SUGAR

FOURNIER, L. Le Sucre Colonial et le Sucre Indigène. 1839.

BY-LAW AMENDMENTS ADOPTED NOVEMBER 18, 1959

Article IV, Section 5 (Board of Managers)

Nominees for Managers to be elected at the Annual Meeting shall be chosen from the voting members of The Institute. Each nomination paper must be signed by at least two voting members of The Institute who shall certify that the candidate will serve if elected. All nominations for Managers made other than by the Nominating Committee shall be delivered to such Committee at least fifteen days before the Stated Meeting of The Institute in the month of December to enable such Committee to consider the same before making its recommendations in accordance with the provisions of Article VIII, Section 3. After the nominations are closed, the presiding officer shall appoint three voting members, who are neither officers nor nominees, to act as tellers of election. The list of nominees shall be incorporated (with directions for voting) in a ballot to be sent by the Secretary to each member entitled to vote, at least ten days before the date of the Annual Meeting. Each ballot shall be provided with a space for the signature of the member voting and shall be accompanied by a return envelope addressed to the Tellers of Election.

Article VIII, Section 3 (Standing Committees of the Board)

The Nominating Committee shall recommend names of nominees for Manager to be presented to the members of The Institute for their consideration in accordance with Article IV, Section 5.

In presenting to the Stated Meeting of The Institute in December its recommendations of nominees for Manager, the Nominating Committee shall also present to said meeting the name of any nominee proposed for Manager by members in accordance with the provision of Article IV, Section 5, if such name does not appear on the list of names recommended for Manager by the Nominating Committee.

At other times as may be necessary, this Committee shall present nominations to the Board of Managers to fill vacancies on the Board or among The Institute's officers, as provided in Article IV, 8 and Article VI, Section 2, respectively.

The names of nominees for officers shall be presented to the organization meeting of the Board, in accordance with Article VI, Section 2.

LABORATORIES' REPORTS AVAILABLE FROM OUTSIDE SOURCES

The following is a selected list of Franklin Institute Laboratories' reports available from the Office of Technical Services or the Library of Congress. Items marked with an asterisk are available from the Office of Technical Services, U. S. Department of Commerce, Washington 25, D. C. The others can be obtained from the Photoduplication Service, Publication Board Project, Library of Congress, Washington 25, D. C. (AD numbers refer to listings by ASTIA (Armed Services Technical Information Agency).)

	Price per copy	Micro-film	Photo-copy
Air Traffic Control			
PB 127837			
Analytical and simulation studies of terminal-area air traffic control. May 1955. 59 p.		$3.60	$9.30
PB 111367			
Analytical and simulation studies of several radar-vectoring procedures in the Washington, D. C., terminal area. April 1954. 70 p.		3.25	9.00
PB 113173			
Communication measurements at the Langley Air Force Base during November and December 1951. November 1952. 89 p.		3.75	11.25
Chemistry			
PB 151521*			
Mechanism and kinetics of the reaction between fuming nitric acid and/or its decomposition products and gaseous hydrocarbons. January 1958. 36 p. AD 155550.	1.25		
PB 131415*			
Mechanism and kinetics of the reaction between fuming nitric acid and/or its decomposition products and gaseous hydrocarbons. June 1957. 62 p. AD 118105	1.75		
PB 121030*			
Study of the absorption spectra and ignition limits of exploding mixtures of carbon disulfide and oxygen. March 1956. 115 p.	3.00		

	Price per copy	Micro-film	Photo-copy
Colloids			
PB 140242			
Development of anti-seize compound for tank application. April 1958. 20 p. AD 200075.		2.70	4.80
PB 97897			
Physical testing of asphalts and similar materials. February 1949. 29 p.		2.00	3.75
Controls			
PB 134859			
Non-linear aspects of angular rate gyroscope systems. December 1953. 169 p.		7.80	25.80
Electronics-Computers			
PB 132255			
Four channel noise source. April 1955. 17 p.		2.40	3.30
Engineering Psychology			
PB 140052			
Visual search in an unstructured visual field. June 1958. 46 p. AFCRC-TR-59-51. AD 211156.		3.60	9.30
PB 140339			
The mechanical power output of men. January 1958. 86 p. AD 200976.		5.40	15.30
PB 131823*			
Dynamic response of human operators. October 1957. 268 p. AD 110693.	4.00		
PB 124680			
Non-linear approach to human tracking. December 1955. 38 p.		3.00	6.30
PB 134387			
Phase-plane as a tool for the study of human behavior in tracking problems. November 1955. 16 p.		2.40	3.30
PB 117121			
Human frequency response studies. June 1954. 81 p.		4.00	11.50
PB 108296			
Spectral density study of tracking performance. Part 1: Effect of instructions. January 1952. 22 p.		2.00	3.75

	Price per copy	Micro-film	Photo copy
Engineering Psychology (Cont.)			
PB 108385			
Spectral density study of tracking performance. Part 2: Effects of input amplitude and practice. January 1952. 22 p.		2.00	3.75
PB 107342			
Preliminary study of the power-spectrum approach to the analysis of perceptual motor performance. October 1951. 46 p.		2.50	6.25
Friction-Lubrication			
PB 161017*			
A bibliography on gas-lubricated bearings, revised. September 1959. 106 p.	2.50		
AECU 3913*			
A study of the stability of externally pressurized gas bearings. November 1958. 41 p.	1.50		
AECU-3773			
Gas-lubricated bearings. July 1958. 27 p.		2.70	4.80
AECU 3771			
The influence of the molecular mean free path on the performance of hydrodynamic gas-lubricated bearings. June 1958. 32 p.		2.70	4.80
AECU 3150*			
Load capacity and dry friction tests of a water-lubricated thrust bearing. 1954. 23 p.	0.25		
TID 5140*			
Investigation of materials for water-lubricated thrust bearings. 1953. 31 p.	0.25		
General Physics			
PB 151019*			
Early detection of fatigue in aluminum alloys by ultrasonics. May 1958. 45 p. AD 155558.	1.25		
Magnetism			
PB 151262*			
Research on fine-particle permanent magnet materials. April 1958. 56 p.	1.50		

	Price per copy	Micro-film	Photo-copy
Magnetism (Cont.)			
PB 151468*			
Preparation of ferromagnetic fine particles from bimetallic oxalates. April 1958. 11 p. AD 155763.	0.50		
PB 151420*			
Magnetization mechanism of multidomain particles. March 1958. 23 p. AD 155764.	0.75		
PB 123134			
Magnetic and structural properties of precipitating ferromagnetic systems. August 1955. 8 p.		1.80	1.80
PB 118560			
Magnetic and structural properties of precipitating ferromagnetic systems. February 1955. 9 p.		1.80	1.80
PB 118559			
Magnetic and structural properties of precipitating ferromagnetic systems. November 1954. 33 p.		3.00	6.30
Metallurgy			
PB 126111			
Tensile properties of zone-refined iron in the temperature range from 298° K to 4.2° K. September 1956. 49 p.		3.30	7.80
PB 122106			
Zone purification of reactive metals. February 1956. 37 p.		3.00	6.30
PB 111933*			
Study of purification and substructure formation of metals and the properties of high purity metals. September 1955. 30 p.	0.75		
NYO 7165			
Dilatometer for studying the densification of powdered compacts. July 1955. 10 p.		1.80	1.80
NYO 6114			
The structure and properties of iron powders prepared by the carbonyl process. December 1954. 21 p.		2.25	4.00
Nuclear Engineering			
AECU 3629			
Analysis of stresses and deflections in top support grid, PWR reactor. June 1957. 88 p.		4.80	13.80

	Price per copy	Micro-film	Photo-copy
Physics of Metals			
PB 135083			
Magnetic examination of the initial precipitation in a gold-nickel alloy. 1957. 39 p.		3.00	6.30
PB 140630			
Determination of electronic specific heat in Cu-Pd alloys. September 1957. 25 p.		3.60	9.30
PB 135716			
Diffusion of cobalt and chromium in chromite spinel. April 1957. 17 p.		2.40	3.30
NYO 7652			
Diffusion of the elements of the IB and IIB sub-groups in silver. December 1956. 19 p.		2.40	3.30
NYO 7482			
Diffusion of Au in single crystals of Ag. March 1956. 16 p.		2.40	3.30
NYO 7477			
Diffusion in metals. December 1955. 6 p.		1.80	1.80
NYO 7168			
Diffusion of zinc in single crystals of silver. September 1955. 15 p.		2.40	3.30
NYO 6112			
Self-diffusion in zinc. September 1954. 62 p.		3.90	10.80

BOOK REVIEWS

ELEMENTARY DECISION THEORY, by Herman Chernoff and Lincoln E. Moses. 364 pages, diagrams, 6 × 9 in. New York, John Wiley & Sons, Inc., 1959. Price, $7.50.

It was indeed a pleasure to review this book. It is the intent of the authors to present a fairly broad foundation in the elements of decision theory, comprehensible to those with only a high school mathematics background. This aim has been admirably accomplished through the liberal use of illustrations which are clever and attention-holding. At the same time, the use of standard mathematical procedures and notation has been developed in a readily comprehensible fashion. This text should go a long way toward introducing decision theory to those who should realize its existence and importance, but who do not have the sophisticated math background of the engineer or physicist.

Concerning the actual content, the following subjects have their nature well expounded, and will serve to illustrate the scope of the text: frequency distributions and their descriptive statistics, basic probability concepts, the concept and importance of utility, nature of strategies in uncertainty models, and the development of strategic models.

Congratulations to the authors on a job well done.

A. J. POLANECZKY
The Franklin Institute Laboratories

MASERS, by J. R. Singer. 147 pages, diagrams, 5¾ × 9 in. New York, John Wiley & Sons, Inc., 1959. Price, $6.50.

Since Townes and his associates announced the successful operation of their ammonia beam maser in 1954, this new device has excited much interest among electronic engineers. However, although masers have been developed along several lines, few applications to electronic systems have yet been made. This is due in large part to the fact that, in order to understand and use masers, the engineer must learn some modern physics. This knowledge does not come easily.

Professor Singer is a physicist teaching in an electrical engineering department, and so is in an excellent position to extend a helping hand to the engineer. His book is just that—a helping hand, and a strong one—but no reader should expect to be able to design his own maser after reading the book. The reader must turn to the original literature, which is referenced in great detail, and spend many serious hours in collateral reading. Of the numerous references given, more than 100 are to literature of the physicist, while only 26 are to periodicals or texts of an engineering nature.

The text covers the fundamentals of induced emission and absorption, gas masers, electron paramagnetic resonance in solids, the two level solid state maser, the three level cavity maser, and traveling wave masers. Several brief appendices give more detail on the three level device. For each type of device, typical structures are described and ranges of operating parameters are given. Their special characteristics and particular applications are noted.

This is a valuable book and should be in the hands of every electronic engineer who has a serious interest in masers. Our only criticism is that it is too brief, and in some spots shows signs of hurried preparation.

A. C. BYERS
The Franklin Institute Laboratories

CIRCUIT THEORY OF LINEAR NOISY NETWORKS, by Hermann A. Haus and Richard B. Adler. 79 pages, diagrams, 6 × 9 in. New York, John Wiley & Sons, Inc.; Cambridge, The Technology Press, 1959. Price, $4.50.

The Technology Press of the Massachusetts Institute of Technology has begun a series of Research Monographs, and this is the second to be published. The idea behind this series is to bring before workers in a particular field certain valuable work which is not of such magnitude as to yield a book. Monographs seem more popular in Europe than in this country where much of the important research lies in reports which may have narrow distribution. The importance of low-noise amplifiers makes this a most timely contribution.

In the reported researches the authors concern themselves with amplifiers—not the design of amplifying devices, but the characterization of their noisiness. Here the concept of "spot-noise" is introduced and refers to the effect of the amplifier upon the single-frequency signal-to-noise ratio. The work thus develops some interesting features relating power and noise in linear, multi-terminal-pair networks.

Starting with the impedance description of the network in matrix form, a "characteristic-noise matrix" is developed. The characteristic-noise matrix is then studied by examination of the properties of its eigenvalues. From studies of the latter, a fundamental or "canonical" network is attained through lossless network transformations performed on the original network. Interconnections of linear noisy networks are also examined. An improved definition of Noise Figure which is more applicable to cases involving the negative resistances of linear amplifiers is developed, based upon the exchangeable power rather than the available power. Finally, some optimization techniques are applied to network realization of amplifiers employing the noise-performance criteria previously developed.

These are fleeting words which attempt to convey the penetrating approach displayed in this monograph. It is difficult to do more than recommend this work emphatically to all theoretical workers on the frontiers of low-noise, high-performance communications circuits.

C. W. HARGENS
The Franklin Institute Laboratories

DYNAMICS OF FRAMED STRUCTURES, by Grover L. Rogers. 355 pages, diagrams, 6 × 9 in. New York, John Wiley & Sons, Inc., 1959. Price, $10.25.

In recent years there have been several important structural failures due to the failure of the engineering profession to consider the dynamics of the structure. Besides this, the development of various types of radars requires a careful analysis of the dynamic characteristics of these structures. In this case, a correct solution is absolutely essential for the radar to function properly. In the past, numerous excellent books have been published on the subject of vibrations and dynamics. Some of these have been definitely directed towards the design of machinery and aircraft. These are of particular value to the mechanical and aeronautical engineer. The designer of bridges and buildings has been less fortunate in having access to a ready reference on dynamics as applied to these structures. *The Dynamics of Framed Structures* should fill this need for the practicing engineer as well as the student, and so should experience a wide acceptance.

The dynamic disturbances considered in this book are due to earthquakes, airblasts, and unbalanced machines as applied to framed structures. In Chapter 2, structures with one degree of freedom are considered, such as cantilever beams in different modes of vibration, simple bents, and vibration due to an elastic foundation. In Chapter 3, structures with two degrees of freedom are considered. Several examples of these are bents with different legs and unsymmetrical beams. Chapter 4 deals with structures having many degrees of freedom. An important structural example in this category is the multi-storey shear building. Such factors as joint rotation, yielding of the foundation, and the effect of being loaded beyond the elastic range are considered. Chapter 5 is devoted to the vibration analysis of slender elastic beams under various conditions—simply supported, fixed supported, free ended, fixed ended but guided, and elastically supported. Chapter 6 deals with inelastic beams and refinements to the theory as applied to elastic beams. In Chapter 7, the vibration of beams subjected to disturbances that are a function of time are treated; examples are beams of highway and railway bridges. Chapter 8 deals entirely with continuous beams and is illustrated by a discussion of two-span beams. Chapter 9 deals with single and multi-storey frames. Chapter 10 covers the vibrations of various types of floor systems (such as beam and girder) and flat slab. The last chapter touches on the use of energy methods as applied to structural dynamics.

The book should have a real value to the practicing structural engineer as well as the student. It is recommended as an excellent text for the undergraduate and post-graduate student.

A. O. BERGHOLM
Radio Corporation of America

LINEAR NETWORK ANALYSIS, by Sundaram Seshu and Norman Balabanian. 571 pages, diagrams, 6 × 9¼ in. New York, John Wiley & Sons, Inc., 1959. Price, $11.75.

Books on electrical engineering fall generally into two groups: those written from the standpoint of the students' ability to absorb knowledge, and those which represent a sort of outpouring of all the varied subjects within the author's ken. *Linear Network Analysis* is an example of the latter, primarily because it is composed of a little of everything, flavored with sprinklings of advanced analytical technique. For example, it seems unrealistic to present in the same volume chapters on fundamental concepts, $v = Ri$, and on analytical properties of network functions involving topology.

Knowing the personal interests of the authors, one can understand how the advanced subjects became mixed with the elementary ones. Balabanian has recently completed a book, "Network Synthesis," which leads to the advanced concepts of network theory and realizability. Seshu has authored papers on topological formulas at the University of Illinois. These men have thus conceived a book of graduate level, but it is unnecessarily lengthened by the inclusion of material which is not uniformly scaled to this level. For instance, the problems appear to be more elementary than one would expect to find in a book designed to challenge the graduate student in this field and to demonstrate the fine points. This perhaps was done in a questionable attempt at clarity.

It would be extremely interesting to know how the students react to a course which employs this book. The hope is that they will absorb it thoroughly and have time for some extra problems which the instructor will have to give to force a more penetrating look. This is asking a lot for there are so many phases of the principal subject which seem to be touched too lightly. The old philosophical question may again be raised here, whether it is better to dig deeper to gain competence in the subject or to write a 571-page excursion.

C. W. HARGENS
The Franklin Institute Laboratories

OUR SUN, by Donald H. Menzel. Revised edition, 350 pages, illustrations, 6½ × 9½ in. Cambridge, Harvard University Press, 1959. Price, $7.50.

Within the past few years several books have appeared on the sun. There has been the revision of Abetti's classic, "The Sun"; M. A. Ellison published his "The Sun and Its Effects" and one of a four volume series "The Sun" edited by Gerard P. Kuiper has put in an appearance. There is merit in every one of these books with each treatment pointing up a particular facet of solar study. However, at this time so much solar research is being undertaken it is inevitable that the later the book the better it presents the current thinking.

Our Sun is written by a ranking authority on the number one star. Dr. Donald H. Menzel, Director of the Harvard College Observatory, began his researches on the sun in the late 1920's. While his studies encompassed all of astronomy, his outstanding and most productive work has been on the sun. Under his guidance, superb solar equipment has been built and put into operation. Truly, Dr. Menzel can be called, "the sun's best friend."

The first edition of this book—one of the Harvard Astronomical Series—appeared in 1949. In the intervening 10 years much has happened. New techniques, new concepts, new advances in allied sciences have contributed to the increase in knowledge of the sun. The introduction of rockets to the study of the solar phenomena has filled some of the gaps in the astronomer's picture of the sun. Even radio astronomy, a new, powerful, probing tool, has been used on this nearest star to unravel some of the mysteries.

The whole of these contributions to our knowledge of the sun is contained in this latest book. With characteristic thoroughness and balance Dr. Menzel has broken up his book into sections to provide coverage for every facet of the sun. Typically, the excellent introduction is historical in background and content and provides not only the early concepts of the sun, but introduces the early sequences of size and distance determinations. Chapter 2 provides details of the solar surface.

Dr. Menzel then displays his technical virtuosity. He discusses Light Atoms and Test Tubes; The Message of Sunlight, and Solar Chemistry. These three chapters comprise the finest exposition of the book. The author has the happy faculty of explaining in a lucid manner abstruse concepts which often form the road block to intelligent understanding.

In his chapter on sunspots it is disclosed that sunspots, far from being the stormy, disturbed areas, are really quiescent patches on the stormy surface of the sun. This is such a surprising fact and has had such little coverage that the author could have explored this further to render fuller explanation.

The next three chapters deal with the surface of the sun. His chapter on the corona is a classic in his treatment of coronal temperatures.

Solar Eclipses, The Energy of the Sun, The Sun's Position in the Universe, Solar Power, and finally, Earth-Sun Relationships complete the book.

Dr. Menzel has produced a stimulating book written in a captivating style acceptable to both layman and professional alike. He writes with the experience which comes from long association with his subject. *Our Sun* is a welcome contribution to scientific literature that will by its authority find its way to the shelves of all thinking astronomers. Many others far from astronomy will read it for its manifold merits.

I. M. LEVITT
The Franklin Institute Planetarium

PUBLICATIONS RECEIVED

GEOLOGY FOR ENGINEERS, by Joseph M. Trefethen. Second edition, 632 pages, diagrams, 6 × 9 in. Princeton, D. Van Nostrand Co., Inc., 1959. Price, $8.50.

ORGANIC CHEMISTRY, by Ray Q. Brewster and William E. McEwen. Second edition, 401 pages, illustrations, 6 × 9 in. New York, Prentice-Hall, Inc., 1959. Price: $10.00 (trade); $7.50 (for classroom adoption).

CLASSIC PAPERS IN GENETICS, edited by James A. Peters. 282 pages, illustrations, 6 × 9 in. New York, Prentice-Hall, Inc., 1959. Price, $3.95 (paper).

GRASS PRODUCTIVITY, by André Voisin. 353 pages, plates, 6½ × 9¼ in. New York, Philosophical Library, 1959. Price, $15.00.

CURIOUS NATURALISTS, by Niko Tinbergen. 280 pages, illustrations, 5½ × 8½ in. New York Basic Books, Inc., 1959. Price, $5.00.

RELATIVITY FOR THE LAYMAN, by James A. Coleman. Reissued, 127 pages, diagrams, 5 × 7½ in. New York, The Macmillan Company, 1959. Price, $3.50.

PERFORMANCE OF LUBRICATING OILS, by H. H. Zuidema. Second edition, 205 pages, illustrations, 6 × 9 in. New York, Reinhold Publishing Corp., 1959. Price, $7.00.

SOAP BUBBLES, by C. V. Boys. Unabridged and unaltered republication of the last revision, 193 pages, illustrations, 5½ × 8 in. New York, Dover Publications, 1959. Price, $0.95 (paper).

PICTORIAL HISTORY OF PHILOSOPHY, by Dagobert D. Runes. 406 pages, illustrations, 8¼ × 11 in. New York, Philosophical Library, 1959. Price, $15.00.

NOMOGRAPHY, by Alexander S. Levens. Second edition, 296 pages, diagrams, 6½ × 9½ in. New York, John Wiley & Sons, Inc., 1959. Price, $8.50.

UNIVERSITY MATHEMATICS, by Joseph Blakey. Second edition, 581 pages, diagrams, 5½ × 8½ in. New York, Philosophical Library, 1959. Price, $10.00.

PROPERTIES OF MATTER, by F. C. Champion and N. Davy. Third edition, 334 pages, diagrams, 5½ × 8½ in. New York, Philosophical Library, 1959. Price, $10.00.

BOOK NOTES

SOURCE BOOK OF THE NEW PLASTICS, by Herbert R. Simonds. 354 pages, illustrations, 6 × 9 in. New York, Reinhold Publishing Corporation, 1959. Price, $10.00.

Recent significant improvements in established plastics and the 100 most important patents issued in the plastics field last year are discussed in this volume. New plastics, their properties, production, price, applications and selection are also included as well as the seven new primary plastics of 1958. Complete information about polypropylene and polycarbonate is given. Also examined is the early German development of stereospecific catalysts. A section on graft copolymers describes the use of that technique in modifying plastics properties. This volume offers a unique approach to plastics and concerns itself almost exclusively with latest developments. It contains the first and only survey of federal-sponsored research in plastics, made by James Kanegis of the Department of Commerce.

CRYSTAL CHEMISTRY OF SIMPLE COMPOUNDS OF URANIUM, THORIUM, PLUTONIUM AND NEPTUNIUM, by E. S. Makarov. 145 pages, 6 × 9 in., diagrams. New York, Consultants Bureau, Inc., 1959. Price, $5.25.

This text, translated from the Russian by E. B. Uvarov, attempts to correlate crystal structure data on the simplest compounds of the most thoroughly studied actinide elements. Summarized in a compact form are abundant factual data on crystal structures heretofore scattered over numerous literature sources. The author gives full details of the crystalline structures of the polymorphic modifications of the actinides and their simplest compounds, and also considers certain questions of general crystal chemistry. Seaborg's cautious position regarding the parent of the "5f" family of elements is adopted.

ENGINEERING MECHANICS, by Dwight F. Gunder and Derald A. Stuart. 391 pages, 6 × 9 in., diagrams. New York, John Wiley & Sons, Inc., 1959. Price, $7.75.

A clear transition between an approach to engineering mechanics and the newer vector treatment is provided in this volume designed for all engineering students. Writing in an informal style with clear and simple topics, the authors draw upon the reader's background in elementary physics and mathematics. The concept of a *fixed* vector is used consistently throughout the statistics sections. In each subtopic of mechanics, new problems are introduced to point out the applications of mechanics in a wide variety of classical and modern technological fields. In addition, this text serves as a valuable reference for the engineer who is interested in adding the vector to his equipment.

ANALYTICAL ELEMENTS OF MECHANICS, by Thomas R. Kane. 250 pages, diagrams, 5¼ × 8¼ in. New York, Academic Press Inc., 1959. Price, $4.75.

This is the first of two volumes intended as a course in classical mechanics. The subject matter is presented on a level sufficiently high to encompass the entire range of phenomena to be considered. Vector analysis is used throughout by the author as the best symbolic language available for this. The book is divided into four sections: 1. Vector Algebra; 2. Centroids and Mass Centers; 3. Moments and Couples; 4. Static Equilibrium. Containing very little discursive material—introductory remarks, allusions to topics of philosophical or historical interest, explanations of the physical significance of mathematically defined quantities, etc.—it is not intended as an entire course in mechanics.

PRINTED CIRCUITS, by Morris Moses. 224 pages, illustrations, 5½ × 8½ in. New York, Gernsback Library, Inc., 1959. Price: $2.90 (paper); $4.60 (hard cover).

As a virtual encyclopedia, this handbook on the "how-to-do-it" of printed circuits and

miniature equipment, was written especially for the radio ham, TV and radio service technician and basement experimenter. Wherever possible, practical techniques and methods have been presented. Divided into seven chapters, the text discusses techniques and materials, component selection and application, batteries and power sources, repair of printed-circuit assemblies, and advanced applications.

A GUIDE-BOOK TO BIOCHEMISTRY, by Kenneth Harrison. 150 pages, diagrams, 5½ × 8½ in. New York, Cambridge University Press, 1959. Price, $3.50.

This book is intended as a simple introduction to biochemistry for students, as either a major or elective subject, in a university first degree course. Specialists in other sciences will find it a useful guide to the aims and scope of biochemical inquiry. After a description of the chief branches of research, the subject is developed in a logical way. The nature and sources of cell energy are discussed and the distinguishing feature of living systems—catalysis by enzymes—is then related to energy production, especially through oxidative mechanisms. Photosynthesis in the green plant, upon which most other forms of life depend, is discussed at length. Three chapters are given to the breakdown of different kinds of foodstuffs with particular reference to animal tissues, and new ideas on how cells are able to control their own metabolism are presented.

ENGINEERING THERMODYNAMICS, by D. B. Spalding and E. H. Cole. 375 pages, diagrams, 5¾ × 9 in. New York, McGraw-Hill Book Co., Inc., 1959. Price, $8.50.

A thorough introduction to the basic principles of thermodynamics is given in this text. The important concepts of the science are emphasized and the fundamentals, rather than applications are stressed. Following a review of the history and nature of thermodynamics, the book is divided into four main sections: Introduction to mechanical units; the first law; discussion of engines and their efficiency; and finally, properties of materials. The importance of clear logical development of theory, beginning with careful definitions of all terms, is stressed throughout.

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CURRENT TOPICS

Electronic Space Camera under Development.—A radically new and simple electronic space camera that can take a continuous strip of weather pictures around the world and turn them into television signals for broadcast to the ground, is under development by the Radio Corporation of America for satellite use.

The new camera, based on a combination of television and electronic printing techniques, was described by scientists of RCA's Astro-Electronic Products Division at the convention of the Society of Motion Picture and Television Engineers. The new device is a compact, single unit whose basic elements are a specially-developed RCA electrostatic tape and an electron gun of the type employed in conventional television camera tubes.

In the laboratory model, the elements are built into a glass vacuum tube roughly similar in shape to a mushroom. The "stem" is a television pick-up tube containing the electron gun. The "head" contains the roll of electrostatic tape mounted in an automatic assembly for winding and re-winding during exposure and read-out, somewhat in the manner of standard motion picture film.

According to the RCA scientists, the new camera has several potential advantages for remote viewing of terrestrial clouds or the moon's surface from space, such as: extreme simplicity and durability, in contrast to systems combining television cameras and magnetic tape storage; very large picture capacity, based on the ability to erase and re-use the tape after each passage, in contrast to photographic film; reduced sensitivity to radiation effects in space; and widely variable speed of operation, ranging potentially from a few pictures per orbit up to picture-taking speed comparable to the 16-per-second rate of 8-mm. motion picture film.

The device can be designed to carry various quantities of tape up to several hundred feet; for example, a continuous strip of pictures showing cloud formations completely around the Earth could be contained in 180 ft. of tape—an amount that could be carried in a camera only slightly larger than present laboratory test models, Moreover, the complete photographic record could be read out of the system and transmitted to the ground within the 5 to 10 min. during which the satellite was within range of a particular station.

Installed in a satellite or space vehicle, the camera unit would be associated with a lens system of a type used in conventional photography. A low-power transmitter would broadcast the picture information to Earth.

The tape consists of thin layers of conducting, photo-sensitive, and insulating materials on a transparent base, giving it an appearance similar to that of motion picture film. Before each exposure, the area of tape unrolled before the lens is given a uniform electrical charge by "flooding" from the electron gun at the rear of the tube. When the exposure is made, the charge is driven from the tape in accordance with the intensity of light in the scene viewed through the lens. The result is an electrical charge pattern on the tape, corre-

sponding to the light pattern of the image. The tape can then be moved ahead automatically for the next exposure, storing the image until it is to be read out. In laboratory tests, images have been stored in this fashion on the tape for up to two weeks without noticeable deterioration.

When the pictures are to be sent to Earth, the automatic winding mechanism is reversed. The exposed tape passes before a scanning beam from the electron gun, which "reads" the electrical charge pattern to produce television-type signals for transmission to the ground station.

The read-out process does not completely destroy the charge image on the tape, so that a picture can be read out several times, if desired, before the information becomes illegible. When the tape is to be re-used, however, the "flooding" beam from the electron gun removes all traces of the previous image, leaving the tape entirely clear for a new exposure.

Chemical Compound for Fighting Fuel and Electrical Fires.—A new chemical compound, considered to be twice as effective as any other extinguishing agent against liquid fuel and electrical fires, has been adopted for emergency use at Army installations. Known as "monobromotrifluoromethane," the compound can also be used to prevent fires by mixing it with helium or nitrogen, thereby creating a combustion-free atmosphere, allowing dangerous jobs to be conducted in safety.

Standardized along with the agent was a special cylinder that weighs less than 3 lb. and holds 1 qt. of the compound. The container features a specially designed nozzle that can discharge the fluid in from 10 to 12 sec.

Developed by the Army Engineer Research and Development Laboratories, Fort Belvoir, Va., the new agent can be used in temperatures as low as 65 deg. below zero and will be particularly helpful in the Arctic, where fire is a crucial problem. Nontoxic and non-corrosive, it can also be used safely in homes.

Tests are now under way on 15- and 50-lb. containers which were also developed for use with the agent. The three new types will eventually replace fourteen sizes now used with carbon dioxide and carbon tetrachloride. A recharging unit has already been developed by the Laboratories for use with the three sizes.

Copper Paste for Screen Printing Circuits on Ceramic.—A new method for producing printed wiring directly on ceramic bases without the use of adhesives has been developed at Bell Telephone Labs. The basis of the new process, which uses standard silk screening techniques for forming the pattern, is a specially formulated copper-bearing paste. Following the printing of the desired pattern on the ceramic base, the piece is fired in a two-step process, resulting in a clean, durable pattern with excellent electrical characteristics.

The paste is prepared from a finely ground mixture of copper oxide and a special glass frit, blended with a standard silk screen printing vehicle. The paste is used to print the pattern on the ceramic, and the "card" is heat-dried to remove solvents. After drying, the card with its pattern is fired in air at 750 C. for 20 min. to burn off the printing vehicle. This operation leaves a non-conducting copper oxide pattern, ready to be reduced to metallic copper.

The second firing operation is conducted at 850 C. for 30 min., in a controlled atmosphere containing hy-

drogen, nitrogen, and oxygen. The hydrogen in the atmosphere reduces the copper oxides to metallic copper, while the oxygen prevents reduction of other oxides in the system and promotes good wetting of the glass frit and the ceramic. Without the oxygen present, a poor bond results.

Printed wiring cards prepared this way can be dip-soldered without bond failure, and without the use of corrosive fluxes. In tests of bond strengths, 20-gauge headed wires were attached to the pattern with an area contact about 0.01 sq. in. Bond failure did not occur until about 2000-psi. tensile pull, and even then generally involved breaking out the ceramic rather than bond failure.

There are other potential uses for the new copper paste in addition to printed circuits. With suitable modification of the vehicle, the copper can be applied with a brush or spray gun. When fired, these coatings form a good base for making metal to ceramic bonds, using lead-tin solders.

Conversion of Salt Water.—The General Electric Company has announced the development of a new process for the conversion of salt or brackish water to pure water. The process is based on the "thin film" concept in heat transfer in which the over-all resistance to heat flow is decreased through the combined use of thin films of water both for evaporation and condensation.

Using an experimental, concentric, vertical tube distillation unit, thin films of water for evaporation are achieved by spreading the fluid on the inner surface of the tube by means of a slowly rotating wiper. In condensing the vapor on the outer surface of the tube, thin films result from the utilizaiton of surface tension. Over-all heat transfer coefficients as high as 8000 Btu./(hr.) (sq. ft.) (deg. F.) have been obtained under laboratory conditions with the experimental apparatus.

Current laboratory activities are involved with the construction of prototype test models to prove the feasibility of applying the "thin film" process to various sizes of demineralization equipment. Based on experimental results, applications of the new process point to potential dollar savings through the reduction in cost of energy consumption, initial cost of equipment, and cost of maintenance.

Computer System Reviews Half-Million Insurance Policy Records in Two Hours.—With the aid of an RCA 501 business data system, the State Farm Life Insurance Company of Bloomington, Ill., soon will be able to review a half million policy records within a 2-hr. period each day.

The RCA 501 stores complete information on a life insurance policy on less than 1 in. of magnetic tape instead of the 14 business machine cards now being used by the company.

The first all-transistor electronic data processing system designed to take over the paperwork loads of business, industry and government, the RCA 501 is being used by insurance companies, manufacturing concerns, public utilities, stock brokerage houses, banks and branches of the Army, Navy and Air Force to simplify and speed up clerical work.

In this high-speed insurance operation, the RCA 501 system will add new records, make changes where called for and post payments. It will also determine whether a premium notice or any other documentary followup is required; if it is, such notices will be turned out by the high-speed printer four times faster than the system previously used.

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